Modeling of Turbulent Flow over 2D Backward-Facing Step Using Generalized Hydrodynamic Equations

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Abstract. The Generalized Hydrodynamic Equations are being investigated for simulating turbulent flows. They were derived from the Generalized Boltzmann Equation by Alexeev (1994), which itself was obtained from first principles via a chain of Bogolubov kinetic equations and considers particles of finite dimensions. Compared to the Navier-Stokes equations, the Generalized Hydrodynamic Equations include new terms representing temporal and spatial fluctuations. These terms introduce a timescale multiplier denoted by τ, and the Generalized Hydrodynamic Equations reduce to the Navier-Stokes equations when τ equals zero. The nondimensional τ is calculated as the product of the Reynolds number and the squared ratio of length scales, $\tau = Re \times (l/L)^2$, where l represents the apparent Kolmogorov length scale and L denotes a hydrodynamic length scale.
In this study, 2D turbulent flow over a Backward-Facing Step (BFS) with a step height of H=L/3 (where L is the channel height) at Reynolds number Re=132000 was investigated using finite-element solutions of the GHE. The results were compared to Direct Numerical Simulations (DNS) utilizing the Navier-Stokes equations, and to a $k - \varepsilon$ turbulence model, as well as experimental data. The comparison encompassed velocity profiles, recirculation zone length, and the velocity flow field. The obtained data confirm that the GHE results are in good agreement with the experimental findings, while other approaches diverge significantly from the experimental data.

Keywords: Turbulent flow · Generalized Hydrodynamic Equations · DNS · Navier-Stokes equations · $k - \varepsilon$ turbulence model · Numerical solution · Comparison with experimental data.

1 Introduction

The Generalized Hydrodynamic Equations (GHE) were derived by Alexeev in 1994 from the Generalized Boltzmann Equation. The Generalized Boltzmann Equation itself was obtained from first principles through the Bogolubov kinetic equations chain, taking into account particles with finite dimensions [1].

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DOI: 10.1007/978-3-031-63783-4_6

1.1 Generalized Boltzmann Transport Equation (GBE)

The kinetic theory of gases is founded upon the solution to the Boltzmann transport equation, governing the space-time evolution of the particle velocity distribution function, F, expressed as

$$\frac{Df}{Dt} = J, \tag{1}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{v}} \tag{2}$$

represents material derivative in space, velocity space and time. J denotes the collision integral ([2], p.11), where $\mathbf{v}$ and $\mathbf{r}$ are the velocity and the radius-vector of the particle, respectively, and $\mathbf{F}$ is the force acting on the particle.

The standard Boltzmann transport equation takes into account the changes in distribution function f on hydrodynamic and mean time between collision scales of infinitesimal particles. Accounting for a third time scale, associated with finite dimensions of interacting particles, gives rise to an additional term in the Boltzmann transport equation resulting in a generalized form given by

$$\frac{Df}{Dt} - \frac{D}{Dt}(\tau \frac{Df}{Dt}) = J, \tag{3}$$

where τ is the timescale, a material property, that Alexeev (1994) related to the mean time between particle collisions. The new term is thermodynamically consistent; more details on the GBE are provided in Alexeev's book [2].

2 Generalized Hydrodynamic Equations as Governing Equations

Hydrodynamic equations can be obtained from Eq. (3) by multiplying the latter by the standard collision invariants (mass, momentum, energy) and integrating the result in the velocity space. These equations are valid for incompressible viscous flow. They have the following form and were originally presented in [5]:

$$\frac{\partial \mathbf{V}}{\partial \mathrm{t}} + (\mathbf{V}\nabla)\mathbf{V} - Re^{-1}\nabla^2\mathbf{V} + \nabla \mathrm{p} - \mathbf{F} = \tau \left\{ 2\frac{\partial}{\partial \mathrm{t}}(\nabla \mathrm{p}) + \nabla^2(\mathrm{p}\mathbf{V}) + \nabla(\nabla \cdot (\mathrm{p}\mathbf{V})) \right\} \tag{4}$$

while continuity equation is

$$\nabla \cdot \mathbf{V} = \tau \left\{ \mathbf{2}\frac{\partial}{\partial \mathbf{t}}(\nabla \cdot \mathbf{V}) + \nabla \cdot (\mathbf{V}\nabla)\mathbf{V} + \nabla^{\mathbf{2}}\mathrm{p} - \nabla \cdot \mathbf{F} \right\} \tag{5}$$

where $\mathbf{V}$ and p are nondimensional velocity and pressure, $Re = V_0 L_0/\nu$ - the Reynolds number, V_0 - velocity scale, L_0 - hydrodynamic length scale, ν -

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kinematic viscosity, $\mathbf{F}$ is a body force and a nondimensional $\tau = \tau^* L_0^{-1} V_0 = \tau^* \nu / L^2 Re$. The terms containing τ are called the fluctuations (temporal and spatial) [1].

We made the following assumptions deriving Eq. (4, 5): (i) τ is assumed to be constant, (ii) the nonlinear terms of the third order in the fluctuations, and the terms of the order τ/Re, are neglected, so the focus is on large Re numbers, (iii) slow flow variation is assumed, neglecting second derivatives in time.

Additional boundary conditions on walls require fluctuations to be zero. The following boundary condition for pressure on walls is expressed as:

$$(\nabla \mathrm{p} - \mathbf{F}) \cdot \mathbf{n} = 0, \tag{6}$$

where $\mathbf{n}$ is a wall normal.

The equations (4) together with equation (6), and boundary and initial conditions for the velocities, constitute the governing equations that we are going to solve for turbulent flows. The Generalized Hydrodynamic Equations become the Navier-Stokes equations when the timescale τ is zero. No additional turbulent model is involved or used to obtain the solution for turbulent flow.

Recently, an analytical solution of GHE for turbulent flow in channel has been obtained, which compares well with a number of turbulent channel flow experiments for Reynolds number from $Re = 2970$ to $Re = 970000$ [6].

3 Backward-Facing Step Flow Problem

The Backward-Facing Step (BFS) flow is illustrated in Figure 1. The flow progresses from left to right over a backward-facing step of height H. The entrance channel width is $W = 2 \cdot H$, as depicted in Figure 1. The Reynolds number for the channel exit width is Re=132000, as in the experiment [11], or the Reynolds number calculated for the step height H is Re=44000. The inlet velocity is U=1.

3.1 Numerical simulations results

GHE model. The averaged flow field, the horizontal velocity contours, obtained with the GHE model are shown in Figure 1(a). We used a finite element mesh refined near the walls for GHE consisting of 27700 nodes (triangular elements with linear approximation for all variables). The non-stationary problem described by Eq. (4) and (5) was been solved until it reached a nearly quasi steady state, and then the averaging of the velocity field was performed over the time interval $t = [100, 200]$.

The parameter τ^* in the expression $\tau = \tau^* \nu / L^2 Re$ is a material property and is not currently known for the air, which was used in the experiment. We varied the parameter τ to fit the velocity profile as shown in Figure 2. By fitting the velocity profile, we also obtained an excellent recirculation zone length, which compared well with the experiment. A similar procedure was performed in our work [4] for water, where fitting of one velocity profile by varying τ^* resulted

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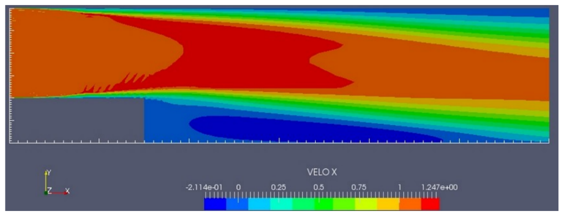
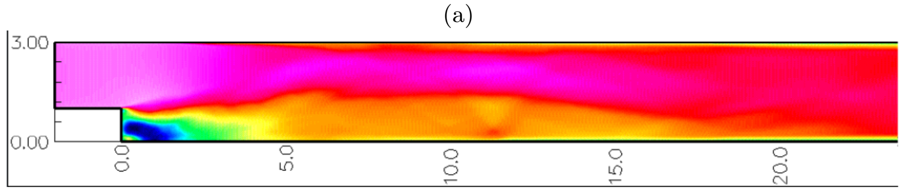


(a)



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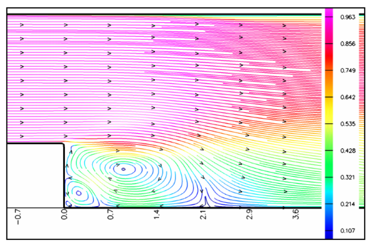

(c)

Fig. 1. Backward-facing step problem: flow progresses from left to right,with the entrance channel width denoted as W and the step height as H=W/3. (a)Contours of the averaged horizontal velocity obtained with the solution GHE by finite-element method at Re=132000, with a mesh of 27700 nodes, (b)Contours of the averaged horizontal velocity obtained with the DNS (Navier-Stokes) solution at Re=132000 with a mesh of 1.1 million, (c) Zoomed-in of streamlines for the DNS (Navier-Stokes) solution.

in excellent agreement for all the velocity profiles across different experiments conducted at various Reynolds numbers. The determined value of τ^* for distilled water remained consistent across different experiments where distilled water was also used.

DNS (Navier-Stokes model). We can estimate how fine the mesh must be to perform a resolved DNS. The smallest scale of the flow is [10]

$$\lambda = \sqrt{\frac{\nu}{|\nabla \mathbf{V}|}}. \tag{7}$$

If we assume that the non-dimensional vorticity is on the order of $Re^{1/2} = 363$ and use $|\nabla \mathbf{V}| = 100$, then we arrive at the estimate $\lambda = 3 \cdot 10^{-4}$. Thus, the grid spacing in the boundary layer (where the vorticity is largest) should be roughly $\Delta s = 3 \cdot 10^{-4}$, and the total mesh size becomes approximately 1.1M grid points.

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Figure 1(b) shows the time-averaged solution when averaged over the time interval $t = [50, 200]$, illustrating the averaged flow field, the horizontal velocity contours. The streamlines of the time-averaged solution, depicted in Figure 1(c), reveal a small recirculation bubble that re-attaches at approximately $x = 2.1H$.

The Overture Cgins module was used for the DNS (Navier-Stokes model) [8], [9].

$K{-}\epsilon$ **model.** The solution with the $k{-}\epsilon$ turbulence model was obtained using the commercial CFD2000 software with a mesh of about 45000 nodes refined at a boundary.

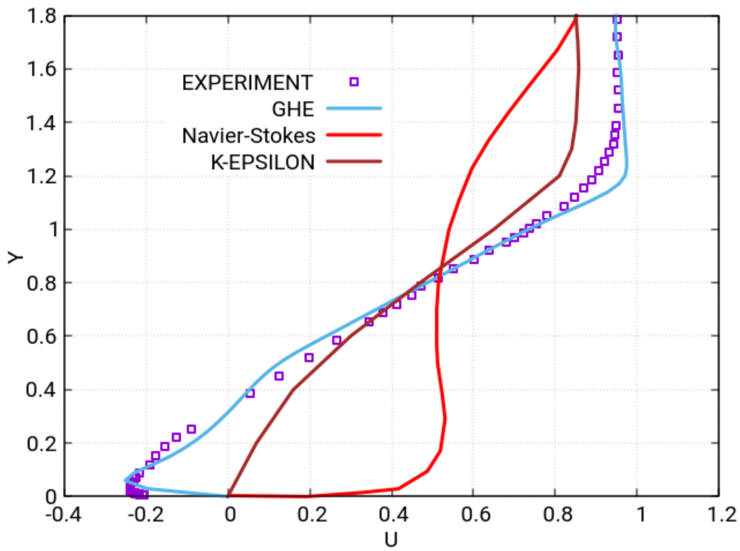

Fig. 2. Flow over a backward facing step, $Re = 132,000$, comparison of the averaged horizontal velocity for the GHE solution, DNS (Navier-Stokes), $k - \epsilon$ model [13], and experimental data (squares) [11] at x = 5.33H.

Comparison of velocity profiles. The flow patterns in Figure 1 are quite distinct. Figure 2 presents the computed velocity profiles at $x = 5.33H$ at the end of the recirculation zone ($x = 0$ at the edge of the step) for different models, along with experimental mean velocity measurements [11].

The solution with a standard $k - \epsilon$ model shows the velocity profile at $x = 5.33H$ that has no backward flow. A standard $k - \epsilon$ model underpredicts the recirculation zone length $X_r = 5.5H$ by a substantial amount, 20-25% according to [12], where more sophisticated turbulence models have been proposed for this problem.

The GHE model output satisfactorily agrees with the experimental data for both the velocity profile and the recirculation zone length X_r. The GHE model

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resulted in $X_r/H = 7.50$, while $X_r^{exp}/H = 7.0 \pm 1.0$ was obtained experimentally. All the data for the recirculation zone length are provided in Table 1.

Model	Recirculation zone length / H
Navier-Stokes	2.1
$k - \varepsilon$	5.5
GHE	7.5
Experiment [11]	7.0 ± 1.0

Table 1. Comparison of the recirculation zone length for different models and experiment.

The results obtained with the DNS (Navier-Stokes equations) show the velocity profile at $x = 5.33H$ without any backward flow, and $X_r/H = 2.1$. Similar conclusion made by Jiang (1993), the Navier-Stokes cannot match the experimental data for $Re \geq 450$, as noted in his NASA report [7]. Experiments by Amaly (1996) show that deviation of the Navier-Stokes results from the experiment begins at $Re = 350$ [3]. Additional equations from a turbulence model need to be added to the Navier-Stokes equations, and these models typically have two to five parameters that need to be tuned for specific problems. In contrast, the GHE does not require a turbulence model.

4 Conclusions

The GHE model was used to simulate 2D flow over backward-facing step at Reynolds number Re=132000 and demonstrated excellent agreement with the experimental data. Unlike other methods, the GHE model does not require any turbulence models, yet it yielded turbulent velocity profiles that matched well with the experimental data (Kim, 1980). This outcome marks a significant improvement over our previous results presented at the ICCS 2010 conference for this type of flow [4], achieved by utilizing a larger number of nodes in the finite element mesh. In contrast, both DNS (Navier-Stokes equations) and the $k - \varepsilon$ turbulence model results for this turbulent flow deviated considerably from the experimental data.

Acknowledgments

Author would like to extend the sincere thanks to W.D. Henshaw for the DNS (Navier-Stokes equations) solution using his Overture Cgins software.

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DOI: 10.1007/978-3-031-63783-4_6