Quantum variational algorithms for the aircraft deconfliction problem

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Abstract. Tactical deconfliction problem involves resolving conflicts between aircraft to ensure safety while maintaining efficient trajectories. Several techniques exist to safely adjust aircraft parameters such as speed, heading angle, or flight level, with many relying on mixed-integer linear or nonlinear programming. These techniques, however, often encounter challenges in real-world applications due to computational complexity and scalability issues. This paper proposes a new quantum approach that applies the Quantum Approximate Optimization Algorithm (QAOA) and the Quantum Alternating Operator Ansatz (QAOAnsatz) to address the aircraft deconfliction problem. We present a formula for designing quantum Hamiltonians capable of handling a broad range of discretized maneuvers, with the aim of minimizing changes to original flight schedules while safely resolving conflicts. Our experiments show that a higher number of aircraft poses fewer challenges than a larger number of maneuvers. Additionally, we benchmark the newest IBM quantum processor and show that it successfully solves four out of five instances considered. Finally, we demonstrate that incorporating hard constraints into the mixer Hamiltonian makes QAOAnsatz superior to QAOA. These findings suggest quantum algorithms could be a valuable algorithmic candidate for addressing complex optimization problems in various domains, with implications for enhancing operational efficiency and safety in aviation and other sectors.

Keywords: Tactical Aircraft Deconfliction · Quantum Approximate Optimization Algorithm · Quantum Alternating Operator Ansatz.

1 Introduction

The global COVID-19 pandemic was not enough to stop the long-term trend of increasing demand for aviation services. According to Airports Council International, in 2023 the number of passengers reached almost 95% of the levels from

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2019, and projections indicate a surpassing of the 2019 level in 2024 [1]. Along with this trend, problems with airspace congestion are returning, and the demand for specialized algorithms dealing with airspace management comes back, one of the problems being the tactical aircraft deconfliction.

In literature, aircraft deconfliction, also known as a conflict detection and resolution problem, refers to the natural and common challenge of ensuring appropriate and safe separation among aircraft operating in the same controlled airspace. The problem arises due to the limited airspace and the need to accommodate multiple aircraft at different directions, altitudes, speeds, and planned maneuvers. The problem has been a subject of interest among many researchers within the community. Despite extensive exploration of conflict detection and resolution, numerous models struggled to sufficiently address the challenges of considered problem, as noted in a seminal work by Kuchar and Yang [16]. Then, the work by Pallottino et al. [21] gained much community attention by introducing the velocity change model, which utilizes mixed-integer linear programming (MILP) to allow real-time maneuvering to resolve aircraft conflicts. This approach was further refined by Alonso-Ayuso et al. [3], who incorporated altitude changes, weather conditions and trajectory recovery into the model while maintaining real-time capabilities.

In a separate study [27], Vela et al. concentrated on addressing the problem of future conflicts, which could occur within a timeframe ranging from 15 to 45 minutes, to minimize fuel costs. They reported achieving near-optimal solutions using the MILP approach, incorporating control over both velocity and altitude. Furthermore, Omer [20] observed that air traffic controllers and aircraft pilots do not favor all velocity, heading, and altitude changes. Consequently, he suggested a discretization approach to facilitate easier handling by human operators, resulting in a minor increase in fuel consumption, amounting to a few kilograms.

Instead of employing MILP, some researchers have proposed using nonlinear programming to address the issue of aircraft deconfliction. In their study [7], Cafieri and Durand utilized Mixed Integer Nonlinear Programming (MINLP) as a natural choice to model separation conditions, addressing the problem using only velocity change. The study conducted by Alonso-Ayuso et al. [4] also applied MINLP formulation to solve the deconfliction problem via turn changes. One notable work that builds upon these two approaches and combines them was conducted by Cafieri and Omheni [8]. They suggest initially resolving the problem by adjusting heading angles and subsequently using this solution as a preprocessing step for modifying velocities.

Various other studies have explored the deconfliction problem, considering factors such as stochasticity and three-dimensional space [17], or employing a new method such as bilevel programming [9]. For an in-depth review of research on deconfliction over the past two decades, one should refer to [22].

Given the recent advancements in quantum computing and still persistent challenges in the broad domain of air traffic management, it is not surprising that researchers have been exploring alternative approaches. The initial study

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that focused on the application of quantum computers in aviation was conducted by Stollenwerk et al. [25], who proposed a method to solve flight-gate assignment problem using the D-Wave 2000Q quantum annealer. Using the same device, Stollenwerk et al. [26] addressed the strategic aircraft deconfliction problem by incorporating takeoff delays into wind-optimal trajectories. Additionally, they outlined a simplified model for trajectory modifications proposing pairwise exclusive avoidance or introducing delays between two consecutive conflicts. The D-Wave 2000Q quantum annealer was also used to solve the Tail Assignment Problem [12] in a study presented by Martins et al. [18]. The problem had been addressed also thanks to classical simulation of a universal quantum computer in [28]. Real gate-based quantum hardware, however, was employed to successfully solve only toy instances of flight-gate assignment in [19, 10].

In this paper, we introduce a novel approach to address the tactical aircraft deconfliction problem using gate-base quantum computers. Inspired by the ideas presented in [20], we advocate for conflict resolution through discretized maneuvers. Our main contributions include designing a proper cost Hamiltonian for the Quantum Approximate Optimization Algorithm coupled with the effective relocation of a subset of hard constraints into the mixer Hamiltonian of the Quantum Alternating Operator Ansatz. Furthermore, we establish a connection with our previous research by benchmarking our approach against a widely-used circle problem dataset published by Rey and Hijazi [23], which has been downscaled to align with the capacity of current quantum machines.

The paper is organized as follows. In Section 2, we formulate the problem, both classically and in quantum terms. In Section 3, we show how to use our formulation with existing quantum algorithms. In Section 4, we describe the results, and conclude the paper with future work in Section 5.

2 Problem Representation and assumptions

Let us assume that during the flight, an aircraft must maintain a minimum separation of 5 nautical miles horizontally and 1000 feet vertically from other aircraft, where a nautical mile equals 1852 meters and a foot equals 30.48 cm. A conflict between two aircraft arises when a pair of aircraft violates at least one of these constraints. If a particular conflict is detected and resolved within five to thirty minutes, then we consider the tactical deconfliction. [22]. We further assume that aircraft motion can be described by a sequence of line segments, maintaining a constant speed within each segment and allowing instantaneous speed changes at the beginning of each segment.

2.1 Classical formulation

We present a graphical summary of our approach to the deconfliction problem in Figure 1. The diagram illustrates the key components of our methodology, including the set of proposed maneuvers and the conflict matrix, which is introduced mathematically later in this subsection.

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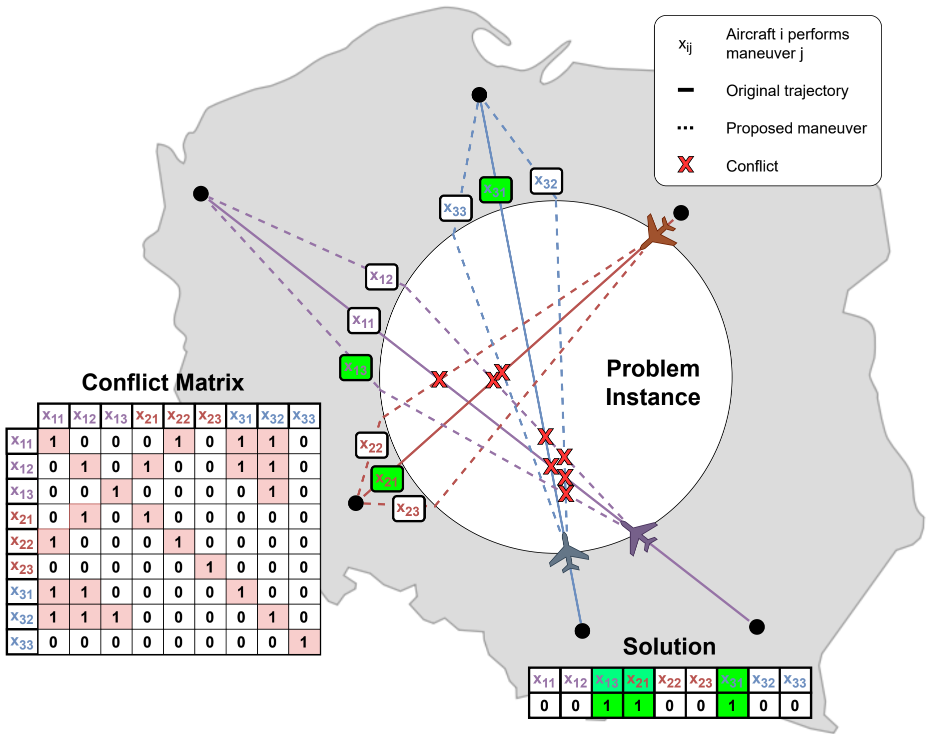

Fig. 1. Diagram summarizing our approach to the deconfliction problem. Initially, three aircraft are in conflict. After proposing 2 additional maneuvers (totaling 3 maneuvers), one feasible solution is proposed: aircraft 1 maneuver 3, aircraft 2 maneuver 1, aircraft 3 maneuver 1. After conflicts are resolved, aircraft may return to their original destinations, which, however, is beyond the scope of our approach.

Given a set of n aircraft with their respective positions, heading angles, speeds, and flight levels, our approach begins by proposing a set of discretized maneuvers for each aircraft. Maneuvers could be of various kinds, including heading angle change, speed change, or flight level change. For simplicity, we assume that each aircraft can perform m maneuvers, although the actual number may vary for an aircraft depending on specific flight requirements. To keep track of these maneuvers let us introduce a set of the following binary variables:

$$X = \{x_{ij} : \ i = 1 \dots, n, \ j = 1, \dots, m, \ x_{ij} \in \{0, 1\}\}. \tag{1}$$

If the variable x_{ij} is assigned the value 1 it indicates that the aircraft i performs maneuver j, whereas a value of 0 indicates the opposite. In this work, we assume that maneuvers are disjoint for an aircraft, i.e., an aircraft must perform one and only one maneuver:

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$$\sum_{j=1}^{m} x_{ij} = 1 \quad \forall i,\ i = 1\ldots,n. \tag{2}$$

After proposing the set of maneuvers for each aircraft, we can then fill a 4-dimensional Conflict Matrix (CM) of size $n \times m \times n \times m$ with binary values indicating presence or absence of a conflict between two aircraft,

$$CM(i,j,i',j') = \begin{cases} 1 \text{ if aircraft } i \text{ performing maneuver } j \text{ conflicts} \\ \quad \text{with aircraft } i' \text{ performing maneuver } j' \\ 0 \text{ otherwise.} \end{cases} \tag{3}$$

To detect the potential conflicts, we use a subroutine proposed by Bilimoria [5] wherein we appropriately transform the coordinate system and calculate the relative aircraft speed. Naturally, the entire matrix is redundant due to its symmetry, i.e., $\mathrm{CM}(i,j,i',j') = \mathrm{CM}(i',j',i,j)$.

The primary focus of the tactical deconfliction problem is to modify aircraft trajectories to resolve all conflicts. This objective can be achieved by satisfying the following constraint:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{i'=1}^{n}\sum_{j'=1}^{m} x_{ij}x_{i'j'}\mathrm{CM}(i,j,i',j') = 0. \tag{4}$$

We can clearly see that, while it is relatively efficient to check whether the solution is feasible, the number of possible solutions grows exponentially with the number of aircraft and maneuvers.

The aircraft deconfliction problem extends beyond the sole consideration of avoiding conflicts as it also encompasses the optimization of various parameters such as fuel consumption or average delay. Typically, such criteria can be aggregated into a cost function to minimize, comprising partial costs for each aircraft:

$$C = \sum_{i=1}^{n}\sum_{j=1}^{m} C_{ij}. \tag{5}$$

In this work, we simplify the optimization process by focusing solely on minimizing the total number of changes to the original trajectory. Nevertheless, the objective can be easily expanded to incorporate more sophisticated criteria as needed.

2.2 Quantum formulation and encoding

When addressing optimization challenges, quantum computing offers a variety of approaches to choose from [2]. In this study, our emphasis is on two different optimization algorithms, namely the Quantum Approximate Optimization Algorithm (QAOA) [11] and the Quantum Alternating Operator Ansatz

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(QAOAnsatz) [14]. These two algorithms are rooted in the Adiabatic Theorem [6], which states that a quantum system in an eigenstate undergoing slow enough changes will remain in that eigenstate. The mathematical connection between these algorithms and the Adiabatic Theorem is not rigid. In practice, the process begins with an arbitrary state, preferably an easy-to-prepare ground state [15]. This initial state then evolves into the ground state that corresponds to the solution of the problem described by the problem Hamiltonian. The subsequent discussion outlines how to construct such a Hamiltonian.

For the translation of the formulas derived in Section 2.1 to quantum Hamiltonians we employ the composition rules described in [13]. In this process, we make use of the Pauli matrices: $I = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, $Z = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$. The first constraint, ensuring that an aircraft can perform one and only one maneuver, can be described in the following way:

$$H_1 = \sum_{i=1}^{n} I - \sum_{j=1}^{m} \left(H_x(x_{ij}) \prod_{j'=1, j' \neq j}^{m} (H_{\text{not}}(x_{ij'})) \right). \tag{6}$$

The Hamiltonian term $H_{\text{not}}(x_{ij'}) = \frac{1}{2}(I + Z_{ij'})$ represents a boolean clause that has a value of 1 if aircraft i does not perform maneuver j'. The product represents a clause with a 1 if any other maneuver, except j, is not performed. We specify the clause that has a value of 1 if aircraft i performs maneuver j by the Hamiltonian term $H_x(x_{ij}) = \frac{1}{2}(I - Z_{ij})$. We repeat the process for every possible maneuver j to achieve a boolean clause that has a value of 1 if we have a correct one-hot encoding. Note that we want the ground state to represent the desired solution, so we must negate the Hamiltonian. Afterwards, we sum over all possible aircraft.

The second constraint, ensuring that no two aircraft are in conflict, is represented as follows:

$$H_2 = \sum_{i,j,i',j' : \text{CM}(i,j,i',j')=1} H_{\text{and}}(x_{ij}, x_{i'j'}). \tag{7}$$

The Hamiltonian term $H_{\text{and}}(x_{ij}, x_{i'j'}) = \frac{1}{4}I - \frac{1}{4}(Z_{ij} + Z_{i'j'} - Z_{ij}Z_{i'j'})$ represents a boolean clause that evaluates to 1 only if aircraft i performs maneuver j and aircraft i' performs maneuver j'. Summing these situations gives us the total number of conflicts. Naturally, our objective is to minimize the number of conflicts, aiming for a value of 0.

The optimization criterion is determined by a Hamiltonian that assigns appropriate weights to the chosen maneuvers of each aircraft:

$$H_{\text{opt}} = \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} H_x(x_{ij}). \tag{8}$$

Here, w_{ij} represents the cost associated with aircraft i performing maneuver j. When aiming to minimize the number of changes from the original trajectories,

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the weights for the original trajectories are set to 0, while a positive value is assigned to the weights corresponding to modified trajectories.

These partial Hamiltonians have been crafted to be combined into a final Hamiltonian, where the ground state aligns with our desired deconflicted solution:

$$H = \theta_1 H_1 + \theta_2 H_2 + \theta_{\text{opt}} H_{\text{opt}}. \tag{9}$$

In the final Hamiltonian, we introduced additional multipliers to ensure that the ground state consistently corresponds to a feasible solution, regardless of the number of changes needed in the original trajectory. A simple valid assignment can be made as follows: $\theta_1 = 1,\ \theta_2 = 1,\ \theta_{\text{opt}} = \text{sum(CM)}$, where sum(CM) is the number of all conflicts (all 1s) in the CM.

3 Application

The two algorithms, QAOA and its enhancement, QAOAnsatz, are both hybrid quantum-classical variational algorithms. In these approaches, a parametrized quantum circuit is designed, and the variational parameters are iteratively adjusted using a classical optimizer to minimize the cost function defined by the expectation value of a chosen observable. We provide a brief overview of the foundations of each of these algorithms and their application in solving the tactical deconfliction problem.

3.1 Quantum Approximate Optimization Algorithm

Given R qubits, QAOA initializes by preparing the quantum register in the state $|+\rangle^{\otimes R}$, which is the ground state of a mixing Hamiltonian composed of Pauli-X gates, $H_M = \sum_{i=1}^{R} X_i$. It then alternately applies the problem Hamiltonian (also known as the cost Hamiltonian) and the mixer Hamiltonian to the initial state, p times, where p is a positive integer. The number p is also referred to as the depth of QAOA. The evolution of Hamiltonians is parameterized by two sequences of variational parameters, namely $\overrightarrow{\gamma}$ and $\overrightarrow{\beta}$. The former controls H_c, while the latter controls H_m. Combining these elements, the final state $|\psi\rangle$ after evolution is expressed as follows

$$|\psi_p(\overrightarrow{\gamma}, \overrightarrow{\beta})\rangle = e^{-i\beta_p H_M} e^{-i\gamma_p H_C} \dots e^{-i\beta_1 H_M} e^{-i\gamma_1 H_C} |+\rangle^{\otimes R}. \tag{10}$$

The role of H_c is to distinguish our desired problem solution by applying a change in phase to it. In the context of the tactical deconfliction problem, we simply need to set $H_C = H$, see Equation 9. The H_M, on the other hand, aims to amplify the phase increasing the probability of measuring the desired solution. This is achieved by adjusting the variational parameters using a classical optimizer which minimizes the expectation value:

$$\min_{\overrightarrow{\gamma}, \overrightarrow{\beta}} \langle \psi_p(\overrightarrow{\gamma}, \overrightarrow{\beta}) | H_C | \psi_p(\overrightarrow{\gamma}, \overrightarrow{\beta}) \rangle. \tag{11}$$

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The expectation value of the circuit measurement is also commonly known as energy. Minimizing the energy is equivalent to increasing the probability of measuring solution to our problem. It is noteworthy that H_c serves a dual purpose, as it also functions as a cost function in this context.

3.2 Quantum Alternating Operator Ansatz

We can modify the approach by initializing the quantum register with a state that corresponds to a feasible solution (or a semi-feasible solution, such as one that satisfies only one of several constraints). The algorithm then applies H_C as usual to distinguish our desired solution in phase, but the mixer Hamiltonian is used differently. It is designed to provide transitions from one feasible solution to another. This way we explore and search for the lowest-energy solution only within a feasible subspace constrained by the hard constraints of our problem, which is the essence of the QAOAnsatz algorithm [14].

In the context of the tactical deconfliction problem, we have chosen to encode only the one-hot constraint (Equation 6) into H_M. To achieve this, we employ a single-qubit ring mixer defined as follows:

$$H_M = \sum_{i=1}^{n} X_{im}X_{i1} + Y_{im}Y_{i1} + \sum_{j=1}^{m} X_{ij}X_{ij+1} + Y_{ij}Y_{ij+1}. \tag{12}$$

Here, the Y symbol represents the Pauli-Y gate. The term $X_{im}X_{i1} + Y_{im}Y_{i1}$ closes the loop between the last and the first qubit, representing the one-hot encoding for each aircraft.

As we have encoded the one-hot constraint into H_M, we can remove the constraint from H_C:

$$H_C = \theta_2 H_2 + \theta_{\text{opt}} H_{\text{opt}}. \tag{13}$$

However, it's important to note that in the presence of noisy hardware, the evolution may drift away from feasible-only solutions. In such cases, having a redundant term in the cost Hamiltonian might be advantageous. In this paper, we choose to use the full cost Hamiltonian, as formulated in Equation 9.

4 Experimental results

In the proposed encoding, the number of qubits was equal to the product of the number of aircraft and their maneuvers. Consequently, instances with an identical number of variables could differ in the ratio of aircraft to maneuvers. We started our set of experiments by investigating how altering these two factors affects instance difficulty. For this purpose, we introduced a set of instances that require only 12 qubits but feature different numbers of aircraft and maneuvers, and these instances were artificially generated by constructing CM to ensure only one solution exists.

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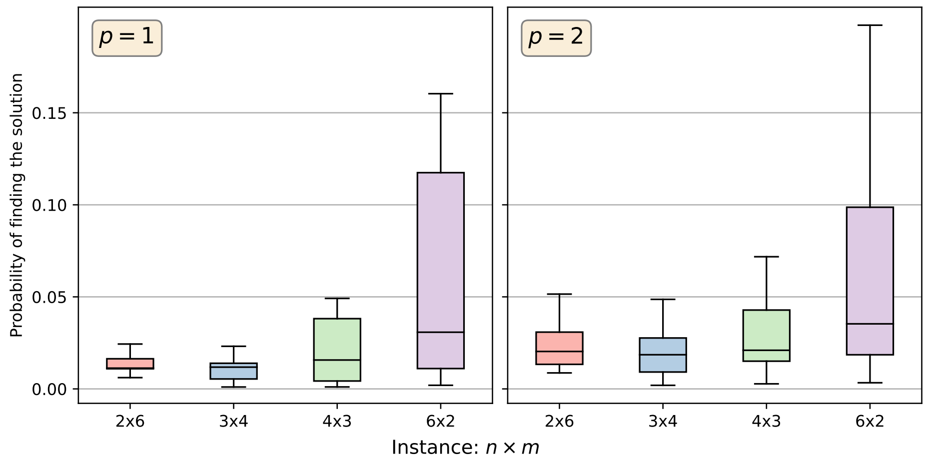

Fig. 2. Average success probability as a function of instance type. Instances are ordered based on the number of aircraft, ranging from 2 aircraft with 6 possible maneuvers to 6 aircraft with only 2 possible maneuvers. The comparison involves two different QAOA depths.

For each instance, we executed 100 QAOA circuits on a noisy simulator with varying initial variational parameters, and the results of success probability were averaged. We used SPSA [24] as the optimizer, as it has proven to perform well on noisy setups. The outcomes are presented in Figure 2. Observing both circuit depths p, we noted that the algorithm faces increasing difficulty in finding the correct solution as the number of maneuvers grows. Conversely, increasing the number of aircraft at the expense of maneuvers tends to make the instance easier. This behavior aligns with our expectations, as ensuring that no two aircraft are in conflict requires less entanglement between qubits compared to constraining that an aircraft can perform one and only one maneuver. More entanglement naturally makes the circuit longer, introducing additional noise. Moreover, entangling gates are typically more error-prone than single-qubit gates. As a side note, we observed that increasing the circuit depth also appears to result in a slight improvement in the average success probability. After conducting initial experiments on a quantum simulator, we evaluated the capabilities of physical quantum computers.

Existing quantum hardware in the noisy intermediate-scale quantum (NISQ) era provides access to several hundred superconducting qubits. Promising qubit implementations use other quantum technologies, such as trapped ions, neutral atoms, or photons. However, the superconducting quantum architectures lack all-to-all qubit connectivity, requiring multiple swaps to make them adjacent before entanglement. Introducing extra SWAP quantum gates may cause additional errors, potentially degrading the solution quality and, in extreme cases, leading to a failure to find one. With this in mind, we decided to downscale

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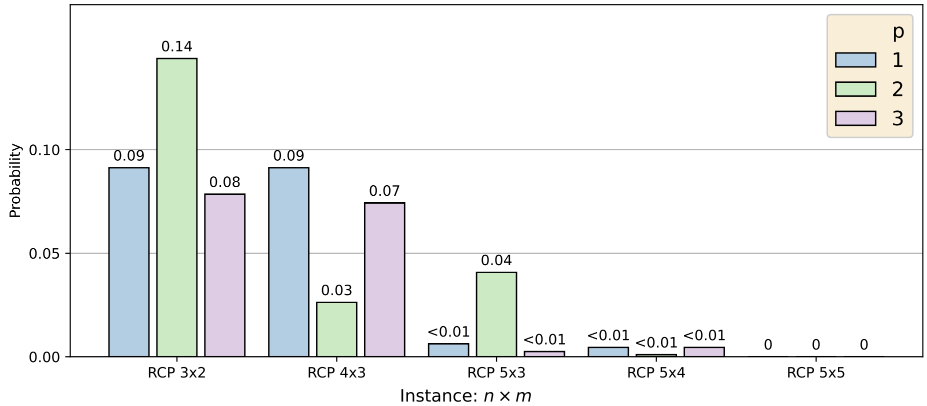

Fig. 3. Probability of finding a solution to the deconfliction problem in the function of instance difficulty and QAOA depth. The instances are the Random Circle Instances with n aircraft, each of the aircraft having m maneuvers to choose from (e.g., $n = 5$, $m = 3$ for RCP 5×3). Experiments were launched on the 133-qubit *ibm_ torino*.

the Random Circle Problem (RCP) instances [23] to involve 3, 4 and 5 aircraft. For instance, with 3 aircraft, we proposed 2 maneuvers and for 4 aircraft we proposed 3 maneuvers. The instance with 5 aircraft was approached with 3, 4, and 5 maneuvers. This results in a total of five RCP instances, requiring 9, 12, 15, 20, and 25 qubits, respectively.

We evaluated the performance of the latest IBM quantum computer using the superconduting 133-qubit *ibm_ torino* quantum computer in solving all instances across three different QAOA depths. The results are illustrated in Figure 3. Clearly, instances requiring fewer qubits are generally easier to solve. As we move to cases with 5 aircraft, the probabilities of measuring a correct solution drop below 0.01 (less than 1%). It is important to note that this low success probability does not indicate failure, as each circuit is typically measured several thousand times. Given the exponential complexity of the tactical deconfliction problem, achieving a correct solution for even a dozen qubits surpasses the performance of a random guess. Even a single positive outcome is sufficient to solve the considered instance. Unfortunately, the quantum computer selected for our experiments could not solve the problem instance with 5 aircraft and 5 maneuvers. Additionally, we could not identify any noticable trend within the circuit depth, largely due to the inherent randomness of a quantum device. Consequently, further experiments are necessary.

Our final set of experiments involved a comparison between QAOA and QAOAnsatz. Once more, we measured the difference on a quantum simulator and take the average of 100 runs. Given that the tactical deconfliction problem is an optimization problem, we choosen to minimize the number of changes to the original flight schedule. Consequently, we present the probabilities of find-

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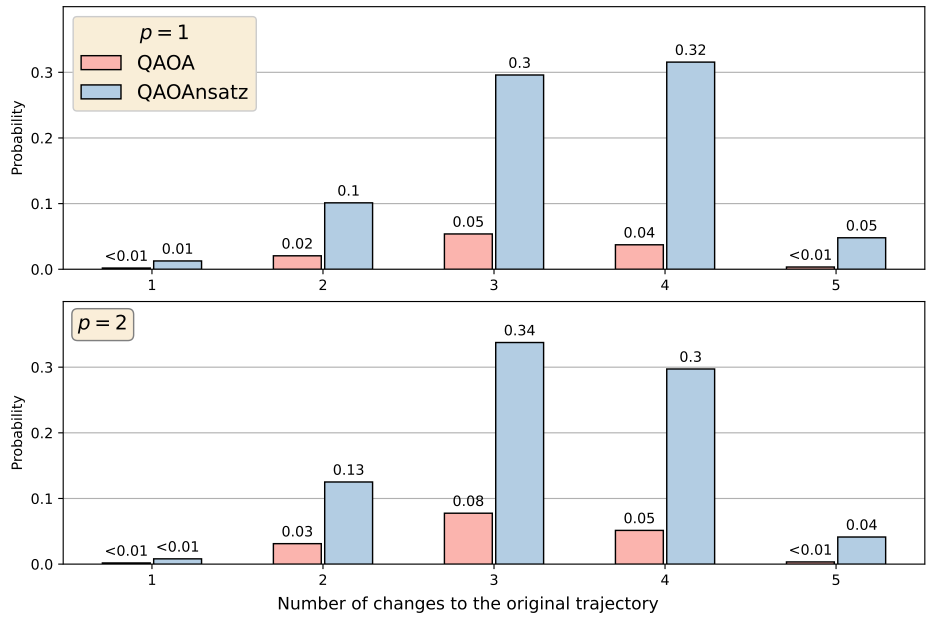

Fig. 4. Comparison between QAOA and QAOAnsatz on a noisy quantum simulator across various two depths, with a focus on the optimization criterion of minimizing changes to the original trajectory. The probabilities of finding a solution to the RCP 5×3 problem are averaged over 100 runs.

ing a correct solution for the RCP 5×3 instance in a function of the number of changes required to achieve a correct solution. The results are shown in Figure 4.

We observed that leveraging the feature of QAOAnsatz, which allows for incorporating hard constraints into the mixer Hamiltonian, provides a significant advantage over using mixers from the vanilla QAOA. The probabilities of measuring a solution to the problem are much higher for QAOAnsatz. However, QAOAnsatz still faces challenges in finding solutions that require only one change to the flight schedule to deconflict aircraft. The experiments demonstrate that QAOAnsatz might be a noteworthy algorithm candidate capable of solving deconfliction instances that QAOA could not handle. We leave this investigation for future work.

5 Conclusions and future work

In this paper, we have successfully shown how to formulate the aircraft deconfliction problem in a way that is applicable to solve using quantum variational algorithms. By designing a proper cost Hamiltonian for the Quantum Approximate Optimization Algorithm (QAOA) and incorporating hard constraints into the mixer Hamiltonian of the Quantum Alternating Operator Ansatz (QAOAnsatz),

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we have demonstrated the efficacy of quantum computing in addressing this challenge. Our experiments have validated the feasibility of quantum approaches in handling the complexity of aircraft deconfliction and shed light on the nuanced interplay between aircraft and maneuvers in determining solution difficulty. Moreover, using physical quantum machines, such as the IBM quantum computer, has underscored the practicality of our proposed methodologies in real-world settings.

We plan to extend our work in a twofold manner. Firstly, we plan to enhance the series of experiments qualitatively. One intriguing avenue for exploration involves investigating the effects of removing the constraint that limits each aircraft to one and only one maneuver. Suppose an airplane can execute more than one maneuver simultaneously. In that case, it implies that both maneuvers are conflict-free, enabling the decision-making process to be deferred to the post-processing phase. Another way of improving the solution finding would be to perform a more in-depth analysis of QAOAnsatz variants, mainly by incorporating controlled state transitions to the mixer Hamiltonian. We should not neglect the fact to address trajectory recovery, which was considered in some papers.

Secondly, we plan to enhance the series of experiments quantitatively by performing more experiments and trying to solve bigger problem instances. Some of the implemented qualitative measures, e.g. moving the one and only one constraint to the post-processing phase, will naturally allow for performing larger experiments. A notable consequence of the time-dependent three-dimensional domain of the problem is that some maneuvers do not with each other. It means that we can find such a bijection between variables and qubits so that the no-conflicting maneuvers correspond to qubits which are distant from each other on the quantum computer processor topology, which could significantly reduce the need for SWAP gates, suppressing the noise. Finally, performing more experiments on the same size instances would also improve the precision and potential findings of the experimental results.

Acknowledgments. This research was supported by Poznan Supercomputing and Networking Center, project no DOB-SZAFIR/01/B/023/01/2021 and partially funded by Poznan University of Technology, project no 0311/SBAD/0734. The access to quantum devices was possible through IBM Quantum Innovation Center established at Poznan Supercomputing and Networking Center.

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DOI: 10.1007/978-3-031-63778-0_22

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