

A novel bandwidth occupancy forecasting method for optical networks

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Abstract. In this contribution, we developed a software tool for collecting information on the data traffic via control plane of an operating optical network. From this data, demand matrix elements were calculated and used to numerically estimate the edge occupancy in the optical network studied. For this purpose, a detailed network model was formulated with cost function and constraints. The formulated network model leads to an optimization problem, which was efficiently solved by meta-heuristic algorithms. Finally, statistical methods were used to model forecasting, in terms of the probability of the edge occupancy, under a Markov process approximation. Additionally, on the basis of the numerical results obtained, the scalability of the applied heuristic and statistical methods was analyzed.

Keywords: Optical network · Modeling · Markov chain · Optimization · Network congestion · Heuristic algorithms.

1 Introduction

Developing an optical network infrastructure is a complex process involving the participation of many stakeholders. Due to the ever-increasing demands for high bandwidth in optical networks, an important need is a rational use of bandwidth and its optimization [7, 1] in Space Division Multiplexing SDM [6, 8], Passive Optical Networks (PON) [14] and in 5G and beyond technologies [2].

The primary stakeholder is the Network Operator (NO), i.e., the company which owns and runs an optical network. NO is interested in minimizing the operational expenditure (opex) and capital expenditure (capex) while maintaining a high level of commercially offered services. NO interacts with optical network

Equipment Providers (EP) when planning Optical Network (ON) maintenance activities and expansion. Before making any decisions on network expansion, the NO needs to have a detailed view of the current use of the available resources and estimates of the future traffic. For this purpose tools are needed that directly monitor the traffic between network nodes, calculate the percentage use of the resources, and estimate the future trends. In this contribution, therefore, we developed a software suite that collects traffic information in real-time directly from the ON via the control plane and creates a database. Then, from the information collected in the database, we derive the elements of the demands matrix and apply optimization methods to minimize capital expenditures and estimate the usage of resources. Unfortunately, when used with standard optimization techniques, the developed methodology results in long computation times [16, 13] including 5G ready networks [12]. Therefore, in this contribution, we propose using specially tailored metaheuristic methods to reduce the calculation time.

In the last step, we perform an analysis and prediction of the capacity resources of the analyzed optical network using statistical methods based on a Markov process modeling with a discrete state space and discrete time ([3], [5], [17], [19]). In particular, the developed software predicts the magnitude of change in the edge occupancy (an increase or decrease) expressed as a percentage or number of occupied DWDM frequency slices in the available band for a given edge in subsequent periods.

The developed software suite consisting of a network traffic monitoring tool, network edge occupancy modeling tool, and the probability of the network edge occupancy forecasting tool presented in this paper is potentially very useful to telecommunication network operators, as it allows for optimal use of the allocated resources and aids the process of network expansion planning. This is because the results obtained allow assessing the need for additional investment into the optical network infrastructure. Hence, the developed model predicts the network edges that are most likely to be subjected to traffic congestion. In practice, operators of large networks pay particular attention to network utilization levels, which are as follows: a safe utilization level is from 0 to 50% (green light), then the range between 50 and 70% is a warning level at which edge expansion should consider (yellow light), while above 70% utilization is at an alarming level (red light), ordering an immediate network upgrade. These assumptions are taken into account in our tests. This allows the network operator to plan in advance the network expansion and allocate appropriate financial resources for the necessary capital expenditure [15].

Taking the above into account, the main contribution of this work is the optimization of network resources using metaheuristics and then, using the results of the optimization, performing an edge-occupancy prediction by applying Markov chain approach (Markov chain states represent the network utilization level). Detailed contributions of this work are the following:

1. network model formulation with cost function and constraints,
2. development of efficient meta-heuristic algorithms for solving the optimisation problem,

3. application of the modeling for prediction the probability of the edge occupancy states using Markov process, and
4. presentation of numerical results evaluating the scalability of the effectiveness of the heuristics and statistical results.

Considering the paper structure, after the introduction, we give the problem formulation. In the 3 Section, we present the proposed metaheuristic algorithms. The 4 Section gives a description of the software tool that collects the data from ON. In the results Section for an optical network, we show the superiority of the proposed metaheuristic approach. Finally, Section 6 provides conclusions and open issues and future work.

2 Problem formulation

An optical network can be modeled by an undirected graph with vertices representing the individual cities and edges as the optical fibers connecting them. The main task of the network is to enable data transmission between all pairs of cities with an expected minimum throughput. The problem is solving the task of deploying enough devices in network nodes to meet the traffic demand while minimizing their cost. This task can be described using mathematical equations derived in [9]. To formulate the problem, the following sets were defined: N - the set of nodes, E - the set of edges, T - the set of transponders, S - the set of frequency slices, $P_{(n,n')}$ - the set of all paths between nodes $n, n' \in N$; $p \subseteq E$, S_t - the set of all frequency slices that can be used as starting frequencies for transponders $t \in T$; $S_t \subseteq S$.

A binary decision variable $x_{tnn'ps}$ equals 1 if a transponder t between nodes n and n' on a path p starting from a frequency slice s is installed and 0 otherwise. For the sake of clarity, it is noted that transponders are installed only at the start and end nodes of a path p . The objective of the optimization problem is to minimize the total cost of transponder installation $\xi(t)$ and the cost of bandwidth usage:

$$\min \left(\sum_{t \in T} \xi(t) \sum_{n, n' \in N} \sum_{p \in P_{(n, n')}} \sum_{s \in S_t} x_{tnn'ps} \right) \quad (1)$$

The optimization is performed subject to constraints. 1. Demands realization between each pair of nodes, 2. Adequate power levels for each optical channel, realized by transponders, and 3. Appropriate and unique allocation of frequency slices for each channel. Their exact mathematical formulation can be found in the literature [10].

Homogeneous Markov chains or Markov Set Chains (if the homogeneity condition was not met) were used to predict the level of slice occupancy.

3 Optimisation Algorithms

To solve this problem presented in Section 2, two metaheuristics based on evolutionary algorithms $\mu + \lambda$ and bee colony (BC) are proposed.

3.1 $\mu + \lambda$ algorithm

The $\mu + \lambda$ algorithm is a basic evolutionary algorithm used to find the best or good enough solution relative to the objective function. It is based on an initial population of μ individuals created randomly. In each iteration of the algorithm, a λ of new offspring solutions are created based on the parent population. Each offspring is created by intersecting the parameters of two randomly selected parents. For each of them, there is a small probability that it will be subjected to a mutation operator that slightly changes its parameters. From the parent population and the offspring population, μ solutions are selected to make up the new parent population. Individuals with a better value of the objective function have a higher probability of being selected for the new population. The algorithm performs successive iterations until a stop condition occurs, usually finding a good enough solution or being permanently stuck in the local optimum. The pseudocode of the $\mu + \lambda$ algorithm is shown in the Algorithm 1.

Algorithm 1 $\mu + \lambda$ Algorithm

```

1: procedure MUPLUSLAMBDA( $\mu$ ,  $\lambda$ )           ▷  $\mu$  - population size,  $\lambda$  - number new
   solutions per iteration
2:    $P \leftarrow$  RandomInitialization( $\mu$ )           ▷ P - population
3:   Evaluate(P)
4:    $best \leftarrow$  ReturnBest(P)           ▷  $best$  - solution with the best goal function value
5:   while stop condition is not met do
6:      $P' \leftarrow$  GenerateOffsprings(P,  $\lambda$ )           ▷  $P'$  - new solutions
7:     Evaluate( $P'$ )
8:      $P \leftarrow$  SelectNewPopulation( $P \cup P'$ ,  $\mu$ )
9:      $best \leftarrow$  ReturnBest( $P \cup best$ )
10:  end while
11:  return  $best$ 
12: end procedure

```

During crossover, the parent with the higher value of the objective function can be favored by increasing the probability p_c that it is its parameters that will be passed on to the created individual. When selecting solutions for a new population, selection pressure is influenced by how strongly we favor the selection of better individuals - it is lowest when the selection is completely random and highest when we select only the best individuals. With higher selection pressure, the algorithm will find better solutions in less time, but it may also get stuck in the local maximum sooner.

3.2 Bee colony algorithm

Another heuristic algorithm used for the study is the bee colony (BC) algorithm. Compared to the $\mu + \lambda$ algorithm, it searches a larger region of the state space and also employs a mechanism to protect against getting stuck in a local maximum.

The BC algorithm is based on an initially randomly generated population of N individuals. During each iteration, new individuals are generated in the following:

1. m of the best individuals are selected from the population. Each of these can be drawn as a parent for a new offspring. A new solution is created based on a single parent - it has the same parameters, but some of them are subjected to the mutation operator. By randomly selecting parents, k_1 new solutions are created;
2. from a population of m individuals, e those with the best value of the objective function are selected. For each of these, k_2 offsprings solutions are created;
3. $N - m$ of new individuals are randomly generated.

Next, N solutions are selected from the initial population, and all the individuals created to form the new initial population. Since the algorithm creates new, completely random individuals in each iteration, the algorithm always has a chance of finding a better solution than the one it encountered in the local maximum. Selection pressure will be determined by the selection methods and the parameters m , e , and k_2 . Decreasing the parameters m and e and increasing the parameter k_2 will increase the selection pressure. The operation of the BC algorithm is shown in the Algorithm 2 pseudocode below.

Algorithm 2 BC Algorithm

```

1: procedure BEECOLONYALG( $N, m, e, k$ )  $\triangleright N$  - pop. size,  $m$  - best bees nb,  $e$  - elite bees nb,
    $k_1$  - new bees created from best bees,  $k_2$  - nb of new bees per one elite bee
2:    $P \leftarrow$  RandomInitialization( $\mu$ )  $\triangleright P$  - population
3:   Evaluate( $P$ )
4:    $best \leftarrow$  ReturnBest( $P$ )  $\triangleright best$  - solution with the best goal function value
5:   while stop condition is not met do
6:      $bestBees \leftarrow$  SelectBest( $P, m$ )
7:      $eliteBees \leftarrow$  SelectBest( $bestBees, e$ )
8:      $bestSol \leftarrow$  RandomNeighbors( $bestBees, k_1$ )
9:      $eliteSol \leftarrow$  EliteNeighbors( $bestBees, k_2$ )
10:     $randomSol \leftarrow$  RandomNeighbors( $P, N - m$ )
11:     $P \leftarrow$  SelectNewPopulation( $P \cup bestSol \cup eliteSol \cup randomSol, N$ )
12:     $best \leftarrow$  ReturnBest( $P \cup best$ )
13:  end while
14:  return  $best$ 
15: end procedure

```

4 Data Collection and Statistical Analysis

As part of presenting the possibilities of utilizing available resources for specific customer connections, data on Performance Monitoring (PM15) was collected every 15 minutes. The study was based on a segment of the Dense Wavelength Division Multiplexing (DWDM) network, belonging to one of the Polish telecommunications operators. The analyzed network segment consists of 7 nodes and 10 edges (E_{xy}), as illustrated by the network diagram in Fig. 1.

The network used in the study relies on NOKIA devices on the 1830 PSS (Photonic Service Switch) platform, and is managed by the Network Management System (NMS). This system enables the management of equipment at the stage of service creation, monitoring the network's state, and collecting network data in real-time. With NMS, it is possible to gather data related to Performance Monitoring (PM). To collect PM15 data for the analyzed nodes, it was necessary to enable the PM15 data collection function for all services at these nodes through NMS. Once activated, PM15 data are recorded in .csv files every two hours and stored on the NMS server for 24 hours. After this period, the PM15 data files are overwritten by data from the next day. This mechanism prevents the server's disk memory from overflowing, so it is important to timely download data files to prevent them from being overwritten and to maintain a continuous record of past network performance.

Four levels of data filtering were adopted, based on PM15 measurements. The first method involves determining four maximum values from a given day for the time intervals: 00:00-06:00 (Time of day 1), 06:00-12:00 (Time of day 2), 12:00-18:00 (Time of day 3) and 18:00-00:00 (Time of day 4). The second method presents a single maximum value from the entire measured day. The third method identifies the maximum value from an entire week. The fourth and final method indicates the maximum value from a given month. Data filtering was conducted using a proprietary program written in Python. Thanks to the application of appropriate criteria, this program is a flexible tool for data analysis.

Specifically, we selected from 7 nodes network a 4 nodes subnetwork consisting of nodes: 1,2,3,4 only and considered traffic exchanged between these nodes only. The data was collected for 365 days 24-hour periods from 18 January 2023 to 17 January 2024 ($\mathbb{T} = \{1, 2, \dots, 365\}$). The input data as a function of time for the one service implemented between nodes 1 and 2 are shown in Fig. 2 - for sampling rate 4 maximum values of the day (Fig. 2a), sampling rate of one maximum value of the day (Fig. 2b), sampling rate of one maximum value of the week (Fig. 2c) and sampling rate of one maximum value of the month (Fig. 2d).

Concerning the application of Markov chain formalism first for each communication channel (i.e. the corresponding time series) between any two distinct network nodes stationarity testing was performed using the Phillips-Perron test with hypothesis H_0 : the time series has a unit root ["no stationary"]. The

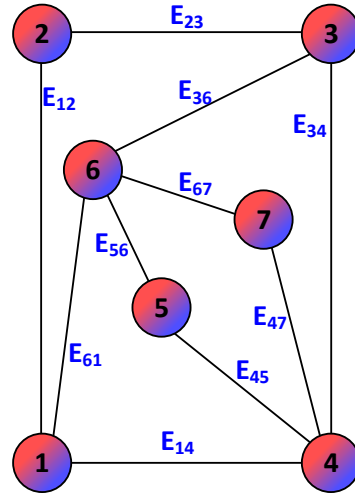
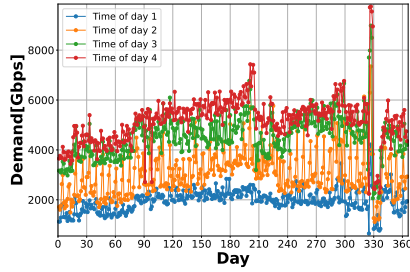
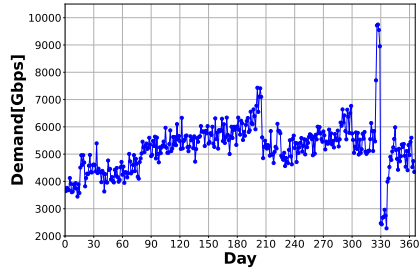


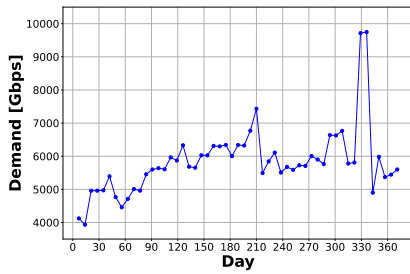
Fig. 1: Diagram of the DWDM network used for the tests.



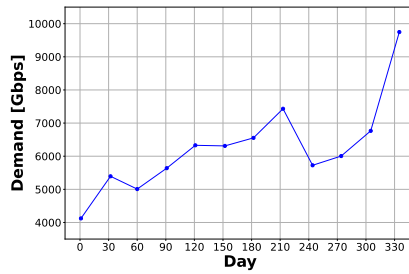
(a) Four maximum values of the day



(b) One maximum value of the day.



(c) One maximum value of the week.



(d) One maximum value of the month.

Fig. 2: Analyzed input data.

test was unequivocally rejected. For H_0 the P-P test yields for service 1: D-F $Z_\alpha = -1410.2$ with p-value < 0.01 and for service 2: D-F $Z_\alpha = -1420.7$ with p-value < 0.01). Consequently, it can be stated that the time series under consideration is stationary. This fact makes it easier to model the series, as it suffices to determine a probability distribution of the time series values, which does not change with time. With this in mind for the analysis of the time series a Markov chain was proposed, i.e., a Markov process with a discrete state space \mathbb{S} and a discrete-time \mathbb{T} , based on a vector of random variables $X_{t_1}, X_{t_2}, \dots, X_{t_n}$ defined on a common state space $\mathbb{S} = \{s_1, s_2, \dots, s_k\}$ and satisfying the Markov condition: $P(X_{t_n} = s_j | X_{t_{n-1}} = s_i, X_{t_{n-2}} = s_{i_{n-2}}, \dots, X_{t_0} = s_{i_0}) = P(X_{t_n} = s_j | X_{t_{n-1}} = s_i)$, which means the transition of the chain from state s_i to state s_j (designation: $p_{ij}(t) = P(X_{t_n} = s_j | X_{t_{n-1}} = s_i)$).

For the purpose of this study, the following state space was defined: $\mathbb{S} = \{s_1, s_2, s_3\}$, where s_1 , denotes "high" level of bandwidth occupancy i.e. above 70% of occupied slices, s_2 , is "medium" bandwidth occupancy and corresponds to between 51% and 70% of all slices used, while s_3 , is "low" bandwidth occupancy with up to 50% of occupied slices in a given edge. The transition probability matrix $P(t)$ was defined as:

$$P(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) & p_{13}(t) \\ p_{21}(t) & p_{22}(t) & p_{23}(t) \\ p_{31}(t) & p_{32}(t) & p_{33}(t) \end{bmatrix}, \quad (2)$$

under the assumptions: $\sum_{j \in \mathbb{S}} p_{ij}(t) = 1, i \in \mathbb{S}$ in the period $(t-1, t]$. Finding a minimum of (1) yields optimal bandwidth occupancy for each time instance, which was then translated into the Markov chain state space \mathbb{S} and treated as microdata n_{ij} , from which the \hat{p}_{ij} were determined ([11], [20]):

$$\hat{p}_{ij} = \frac{\sum_{t \in T} n_{ij}(t)}{\sum_{t \in T} \sum_{j \in \mathbb{S}} n_{ij}(t)}, \quad i, j \in \mathbb{S}. \quad (3)$$

If \hat{p}_{ij} does not depend on time $t \in \mathbb{T}$ (formula (3)), it is assumed that the studied phenomenon is described by a homogeneous Markov chain.

Based on available empirical data, this research considered the following periods: Jan 2023 - Jan 2024, Jan 2023 - Jul 2023 and Aug 2023 - Jan 2024. However, due to the increased network traffic in the second half of 2023 and the space limitations, the rest of the article focuses mainly on the period Aug 2023 - Jan 2024.

5 Results and Discussion

First we apply the proposed heuristic methods to the considered problem of optimizing the frequency band occupation (band slices used) at the edges of an optical network subject to known demands matrix (cf. an example shown in Table 1). The simulation has to be repeated for each time instance considered. So, if we consider 365 samples taken daily over the entire year then the optimisation procedure has to be repeated 365 times. Thus any acceleration of the optimisation calculations is of paramount importance for the analysis performed. Once optimisation simulations are completed we analyse the calculated results using a methodology based on Markov chains.

The results of optimisation performed using heuristic methods have been compared with a deterministic Mixed Integer Programming (MIP) reference method based on integer programming and available through the CPLEX package [4]. The traffic demands (demand matrix elements expressed in Gbps) were calculated using statistical methods as described in Section 4. The calculations were carried out using a linear solver engine of CPLEX 12.8.0.0 on a 2.1 GHz Xeon E7-4830 v.3 processor with 256 GB RAM running under the Linux Debian operating system. Table 2a summarizes the sets used by the optimization procedures, while Table 2b lists modeling parameters for performing computations.

	2	3	4
1	5300	0	0
2	—	0	4450
3	—	—	0

Table 1: Example of a demand matrix from a specific day.

Table 2: Parameters description and fitness function results.

Set	Set settings	Constant	Settings	Analyzed Case	Fitness function
\mathcal{N}	4	bitrate [Gbps]	$v(1) = 10$	day average	MIP BC ($\mu + \lambda$) 205 205 205
\mathcal{E}	4		$v(2) = 100$	week average	215 215 215
\mathcal{S}	384 slots		$v(3) = 400$		
\mathcal{T}	3 transponders	$\xi(t)$	$\xi(1) = 1$	Analyzed Case	Comp. time [sec.] MIP BC ($\mu + \lambda$)
\mathcal{S}_t	$\mathcal{S}_1 = \{1 \dots 384\}$		$\xi(2) = 3$	day average	99.0 0.9 0.8
	$\mathcal{S}_2 = \{1 \dots 382\}$		$\xi(3) = 12$	week average	99.8 0.9 0.8
	$\mathcal{S}_3 = \{1 \dots 380\}$				

(a) Set settings. (b) Constant settings. (c) Fitness and time.

Table 2c shows the values of the objective function for the methods analyzed. The results obtained with the proposed metaheuristics do not differ from the reference MIP method. Table 2c also contains the computation time needed to complete calculations using the proposed metaheuristics ($\mu + \lambda$) and BC and for the deterministic MIP method. An important finding is that the computation time of the proposed methods is much less than that of the deterministic MIP method. The results confirm that the calculation times for the proposed metaheuristics methods are at least two orders of magnitude lower than for the MIP reference method. Such improvement in computational efficiency allows effective implementation of the proposed design methodology too much more complex networks than in the case of MIP. Thus, these results encourage further research on designing realistic networks for large telecom operators using this methods.

Fig. 3 shows bandwidth occupation for edge E21 of the analyzed network as a function of time each day (whole period), calculated by heuristic methods while Table 3 shows the calculated average of the occupation for each edge expressed as a percentage of the total bandwidth. It can be seen that, for the analyzed services, the average result does not exceed the alarming level of bandwidth occupancy (70%). However, results from Fig. 3 show that the alarming level is reached in many time instances of the second half of the year, cf.in particular Fig. 3a and 3d. The warning level (50%) is reached already in the first half of the year. Such results in practice would suggest a telecom operator to think about expanding the network.

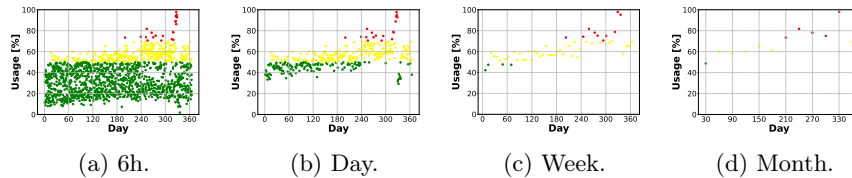


Fig. 3: Edge occupancy (E12) in the analyzed period for different sampling rates.

Edges	MIP slice av.	BC slice av.	$(\mu + \lambda)$ slice av.
	%	%	%
E12	50,40	50,76	50,66
E23	50,00	50,32	50,61
E34	50,00	50,32	50,61
E14	49,20	49,72	50,46

Table 3: Results of the slice occupation.

In the next step we applied Markov chain methods to the results obtained by the heuristic optimisation algorithms. The application of the homogeneous and non-homogeneous Markov chain to the model described in the section 4 for each edge ($E_{12}, E_{23}, E_{34}, E_{14}$) of the graph made it possible to determine the transition probability matrix P (or $P(t)$ given in the (2), depending on the BC algorithm used (P^{BC}) or $\mu + \lambda$ ($P^{\mu+\lambda}$) taking into account the Markov chain running time. In this case, the stochastic process was observed from August 2023 to January 2024, and 6-hour, 1-day, 1-week and 1-month network traffic measurements were used to construct the P matrix. Due to the limited number of pages, the rest of the article includes results only for the E_{12} edge.

Thus, based on (4) - (5):

$$\widehat{P_{6h,E_{12}}^{BC}} = \begin{bmatrix} 0.23 & 0.09 & 0.68 \\ 0.04 & 0.38 & 0.58 \\ 0.03 & 0.32 & 0.66 \end{bmatrix}, \quad \widehat{P_{Da}^{BC}} = \begin{bmatrix} 0.35 & 0.55 & 0.1 \\ 0.1 & 0.73 & 0.17 \\ 0.03 & 0.56 & 0.42 \end{bmatrix}, \quad \widehat{P_{We}^{BC}} = \begin{bmatrix} 0.44 & 0.44 & 0.11 \\ 0.36 & 0.57 & 0.07 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad (4)$$

and

$$\widehat{P_{6h,E_{12}}^{\mu+\lambda}} = \begin{bmatrix} 0.29 & 0.05 & 0.67 \\ 0.05 & 0.36 & 0.59 \\ 0.02 & 0.32 & 0.66 \end{bmatrix}, \quad \widehat{P_{Da}^{\mu+\lambda}} = \begin{bmatrix} 0.28 & 0.61 & 0.11 \\ 0.11 & 0.72 & 0.17 \\ 0.02 & 0.45 & 0.52 \end{bmatrix}, \quad \widehat{P_{We}^{\mu+\lambda}} = \begin{bmatrix} 0.33 & 0.67 & 0 \\ 0.28 & 0.72 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (5)$$

one can determine the probability of transitioning $p_{ij}(i, j \in \mathbb{S})$ between states $\mathbb{S} = \{high, medium, low\}$, as well as staying in one of the states by knowing the initial distribution of d_0 relative to the occupancy of % slices at time t_0 .

For example, if we assume that the matrix $\widehat{P_{6h,E_{12}}^{BC}}$ was generated from state transitions \mathbb{S} for edge E_{12} in the period August 2023 - January 2024, then - assuming the homogeneity of the Markov chain - in 3 periods P will take the form:

$$\left(\widehat{P_{6h,E_{12}}^{BC}}\right)^3 = \begin{bmatrix} 0.048 & 0.321 & 0.644 \\ 0.042 & 0.332 & 0.638 \\ 0.042 & 0.336 & 0.645 \end{bmatrix}. \quad (6)$$

The matrix P obtained in (6) is close to the ergodic matrix E (stationary state) and means that it will remain in the *high* state in period t , provided that in the previous periods $t-1$ the process was also in state *high*.

The ergodic distribution $e = (e_{high}, e_{med}, e_{low})$ for edge E_{12} , with the proposed algorithms for the optimization methods ($BC, \mu + \lambda$) and the accuracy is presented in Table 4.

Edges	BC			$\mu + \lambda$		
	e_{high}	e_{med}	e_{low}	e_{high}	e_{med}	e_{low}
$e_{6hAuJa,E12}$	0.04	0.33	0.63	0.04	0.33	0.64
$e_{DaAuJa,E12}$	0.12	0.67	0.21	0.11	0.64	0.25
$e_{WeAuJa,E12}$	0.33	0.52	0.15	0.29	0.71	0

Table 4: Ergodic distribution e .

Since the Markov chain used for the empirical data described in chapter 4 is irreducible, non-periodic and has an ergodic distribution e , the mean recurrence time r_j to state j can be determined according to the formula: $r_j = \frac{1}{e_j}$. Thus calculated mean recurrence times r_j are presented in Table 5.

Edges	BC			$\mu + \lambda$		
	r_{high}	r_{medium}	r_{low}	r_{high}	r_{medium}	r_{low}
$r_{6hAuJa,E12}$	25.43	3.03	1.59	26.61	3.08	1.57
$r_{DaAuJa,E12}$	8.55	1.49	4.75	9.58	1.56	3.93
$r_{WeAuJa,E12}$	3.0	1.92	6.75	3.4	1.42	Inf

Table 5: The mean recurrence time r_j .

The mean first passage time (expected first return time from state j to i) of Markov chain $M = [m_{ij}]$ where the elements of the matrix M are taken from a recursive formula: $m_{ij} = p_{ij} + \sum_{k \neq j} p_{ik}(m_{kj} + 1)$ and $i, j \in S = \{high, medium, low\}$

Edges	BC		$\mu + \lambda$	
	$medium \rightarrow high$	$low \rightarrow high$	$medium \rightarrow high$	$low \rightarrow high$
$m_{ij_{6h},E12}$	31.1	31.7	34.7	35.9
$m_{ij_{Da},E12}$	11.4	12.6	11.7	13.2
$m_{ij_{We},E12}$	-	-	-	-

Table 6: The Mean First Passage Time $m_{i,j}$.

As mentioned, the original data came from Jan 2023 to Jan 2024. Due to the increased network traffic in the second half of 2023, this period was included in the above analyses. By increasing the frequency of data observation (e.g. every 15 minutes), you can notice that there are periods of increased traffic (e.g. in the evening) compared to other times of the day. In such situations, the condition of the uniformity of the Markov chain may not be met. Then we propose Markov Set Chains (the algorithm for determining the upper e^{HI} and lower e^{LO} bounds for the elements of the ergodic distribution e of the transition probability matrices P_1, P_2, \dots, P_k is included in, among others, the papers: [3], [18]). The values

of the constraints e^{HI} and e^{LO} for the ergodic distribution e of the relevant transition probability matrices for the studied period are included in Table 7.

Taking into account the number of transponders used $N = \{n_{40}, n_{100}, n_{400}\}$, the cost of 40T, 100T, 400T transponders: $pr \in Price = \{1, 3, 12\}$ USD respectively, the ergodic distribution e for the states $S = \{high, medium, low\}$ with respect to the edges $k \in \{E_{12}, E_{23}, E_{34}, E_{14}\}$, it is possible to determine the cost C related to the charge on the paths E_{12}, E_{23}, E_{34} and paths E_{14} : $C_{E_{12}, E_{23}, E_{34}} = \sum_{j \in N} \sum_{i \in S} \sum_{k \in \{E_{12}, E_{23}, E_{34}\}} n_j * e_{ik} * pr_j$ and $C_{E_{14}} = \sum_{j \in N} \sum_{i \in S} n_j * e_{iE_{14}} * pr_j$ where e_{ik} is the i -th coordinate of the ergodic distribution e for the edge k . Assuming that the edge occupancy in both paths should be equal, a measure $M_D = \frac{C_{E_{12}, E_{23}, E_{34}}}{C_{E_{14}}}$ can be proposed, where $D = \{6h, Da, We, Mo\}$ denotes the frequency of network monitoring (M_D closer to 1 indicates the choice of method). In the case of empirical data, the indicator values are: $M_{6h} = \frac{32.886}{10.948} = 3.004$, $M_{Da} = \frac{33.733}{11.246} = 2.999$, $M_{We} = \frac{34.027}{11.341} = 3$, and $M_{Mo} = \frac{34.322}{11.355} = 3.023$ regardless of the algorithm used: $BC, \mu + \lambda$ ($M_{D, BC} = M_{D, \mu + \lambda}$). Taking into account $\min\{M_{6h}, M_{Da}, M_{We}, M_{Mo}\}$, the lowest value is M_{Da} , so the recommended frequency of network traffic monitoring is daily.

Edges	BC			$\mu + \lambda$		
	e_{high}	e_{med}	e_{low}	e_{high}	e_{med}	e_{low}
$\frac{LO}{e_{6h, E_{12}}}$	0.01	0.2	0.53	0.01	0.2	0.52
$\frac{HI}{e_{6h, E_{12}}}$	0.1	0.44	0.75	0.12	0.45	0.75
$\frac{LO}{e_{Da, E_{12}}}$	0	0.12	0	0	0.38	0.01
$\frac{HI}{e_{Da, E_{12}}}$	0.38	0.99	0.82	0.3	0.96	0.51
$\frac{LO}{e_{We, E_{12}}}$	0	0	0	0	0	0
$\frac{HI}{e_{We, E_{12}}}$	1	1	1	1	1	1

Table 7: Markov Set Chains - Ergodic distribution $e_{HI} \geq e \geq e_{LO}$.

Based on the results obtained, the following observations can be made:

1. regardless of the algorithm used, P matrices achieve ergodic distribution relatively quickly (usually after 5-6 periods with an accuracy of 3 decimal places), which is mainly due to the small number of S states,
2. in the case of data from the period Jan'23-Jan'24 and Aug'23-Jan'24, the probability of exceeding 70% is slightly higher using the BC algorithm than $\mu + \lambda$ for data 6h, Da and We without depending on the analyzed edge,
3. in the case of data from Jan'23 - Jan'24, the probability of reaching the *high* state is usually twice lower than in the period Aug'23 - Jan'24. This increased network traffic in the second period, probably dictated by the autumn parliamentary elections, is an indication for the operator to be prepared for a potential increase in network traffic in the event of the next elections (e.g., securing the appropriate number and power of transponders),
4. obtained in the table 5 the mean recurrence time r_j confirms the previous conclusion: the time to return to the *high* state after leaving it is shorter in

the case of data from the period Aug‘23 - Jan‘24 and usually slightly shorter in the case of the BC algorithm,

5. the first passage time to the *high* state from the *medium*, *low* state is usually slightly shorter in the case of the BC algorithm,
6. the previously mentioned significant increase in network traffic in the second half of 2023 may suggest using a non-homogeneous Markov chain. Transition probability matrices $P_{Jan23-Jul23}$ and $P_{Aug23-Jan24}$ verified by the χ^2 compliance test with the $\chi^2_{\hat{P}_{Jan23-Jul23}, P_{Aug23-Jan24}}$ test of $H_0 : \hat{P}_{Jan23-Jul23} = \hat{P}_{Aug23-Jan24}$ differ significantly (e.g. for $Da, \mu + \lambda, E_{12} : \chi^2_{emp} = 127.55, df = 8, p - value < 0.001 \Rightarrow H_0$ rejected). In such a situation, we propose to model the network traffic using Markov Set Chains (HI-LO, [3]), i.e. we determine the intervals in which the proper ergodic distribution is realized: $e_{LO} \leq e \leq e_{HI}$ (results in table 7),
7. based on the results in the 7 table, it can be seen that:
 - with a decrease in the frequency of data downloading, the span of the interval covering the coordinates of the $e = (e_{high}, e_{med}, e_{low})$ increases, which means less precision in determining the estimates r_i and m_{ij} ,
 - HI-LO intervals around e from the period from Aug‘23 to Jan‘24 are usually narrower than those determined based on the period from Jan‘23 to Jan‘24 (similarly in the case of r_{high} - return time to *high* and $m_{i,high}$ - the time of reaching *high* for the first time from state i), therefore it is advisable to use different Markov chains for different periods to model network traffic.
8. Based on the M_* measure, we propose using daily data.

Once we developed the Markov chain formalism, we can use the determined transition probability matrices P to forecast the bandwidth occupancy for 365 days ahead (Fig. 4), based on the ergodic distribution for the state space \mathcal{S} . Consequently, the predicted expected bandwidth usage does not change in time

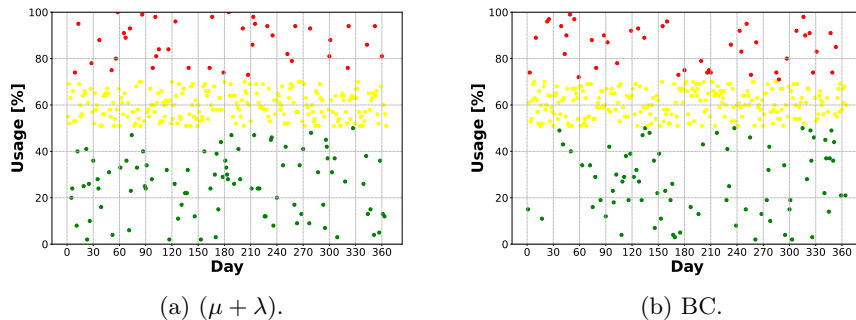


Fig. 4: Forecasted bandwidth occupancy at the edge E_{12} for a year ahead, and two proposed methods.

for each state, which limits the model's accuracy. However, the model is still fully capable of predicting the probability of the state of the particular edge occupancy in the future days, which is the key point illustrated by the results shown in Fig. 4.

6 Conclusions

This contribution presents a methodology for an analysis of bandwidth occupancy in an optical network. The presented approach has been applied to real data collected via a management system from an operating network. For the calculation of bandwidth occupancy within a considered time period optimization metaheuristic methods are used to accelerate the numerical calculations. The results obtained confirm that the calculation times for the proposed metaheuristics methods are at least two orders of magnitude lower than for the MIP reference method. This improvement in computational efficiency allows an effective implementation of the proposed analysis methodology. Finally, the forecasting needed for optical network expansion planning has been performed using the Markov chain approach. The derived Markov transition probability matrices are used to predict traffic in network edges for future days using the data calculated by the metaheuristic methods for the preceding time period. Using the calculated Markov chain transition probability matrices, we have calculated a bandwidth occupancy forecast for a year ahead and thus demonstrated the practical relevance of the presented approach.

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