# SESP-SPOTIS: advancing stochastic approach for re-identifying MCDA models

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**Abstract.** Multi-Criteria Decision Analysis (MCDA) is an interdisciplinary field that addresses decision-making problems that involve multiple conflicting criteria. MCDA methods are widely applied in various domains, including medicine, management, energy, and logistics. Despite their widespread use, MCDA techniques continuously evolve to address emerging challenges. This paper presents a new method called Stochastic Expected Solution Point SPOTIS (SESP-SPOTIS), for re-identifying MCDA models. SESP-SPOTIS conducts a stochastic search for the Expected Solution Point (ESP) which is then utilized within the Stable Preference Ordering Towards Ideal Solution (SPOTIS) framework. The study delves into comprehensive investigations of MCDA model re-identification and examines how the updated model influences the ranking of analyzed alternatives. Furthermore, the experiments were divided into training sets and tests to evaluate the similarity of the proposed approach, using two rank correlation coefficients, namely Weighted Spearman ($r_w$) and Weighted Similarity ($WS$). The results demonstrate that SESP-SPOTIS effectively re-identifies updated models and provides additional information from analysis as an ESP, thereby broadening knowledge and understanding in the decision-making process of the analyzed problem. By integrating machine learning models and stochastic optimization techniques, SESP-SPOTIS contributes to advancing the methodologies for MCDA model re-identification.

**Keywords:** MCDA · Re-identification · SPOTIS · PSO · Reference point

## 1 Introduction

Multi-Criteria Decision Analysis (MCDA) is an interdisciplinary field that solves decision-making problems with multiple criteria, often in conflict with each other. These methods are used in various areas of science and practice, such

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as medicine [10], energy [7], or logistics [24]. The relatively widespread use of MCDA techniques leads to the continuous development of new approaches that respond to emerging challenges. A Rank Reversal (RR) phenomenon occurs when one alternative is removed or added from a decision problem, leading to a change in the ranking of some other alternatives. This is one of the challenges in such classic methods as Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [6] and VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [5].

In 2014, a method known as the Characteristic Objects Method (COMET) was developed to deal with the rank reversal phenomenon by using characteristic objects and fuzzy logic to evaluate alternatives [19]. In 2016, the Reference Ideal Method (RIM) was created to evaluate various decision-making alternatives using a reference point (Reference Ideal) [2]. Like COMET, the RIM method was designed to be robust to the problem of reverse rankings. This was followed in 2020 by the Stable Preference Ordering Towards Ideal Solution (SPOTIS) method, characterized by its simplicity and resilience to reverse rankings [4]. However, these methods shared a common factor with the SPOTIS method because they were also based on reference points. In [4], Dezert et al. proposed, in addition to the classical approach used in many MCDA methods, that decision-makers determine a reference point rather than just inferring based on ideal point alternatives. This point was called the Expected Solution Point (ESP). However, due to such a point, another challenge arises in re-identifying such models.

Re-identification involves trying to re-map an existing model. It is based on a learning procedure to rank the considered decision variants [17]. For this purpose, machine learning models or optimization methods are used allowing to adjust the optimal parameters for MCDA methods to get the model as close to the original one as possible. The process of re-identifying decision models involves training the models. This process aims to find a new set of parameters that minimize prediction error or maximize the similarity of rankings. In this way, it is possible to effectively rebuild an unknown model to evaluate previously unconsidered alternatives in the context of a given decision problem. In addition, retrieving the lost parameters of the decision model, such as the weights of the criteria, will allow a more accurate analysis of the results obtained.

There are several previous works in which the focus was on the re-identification of MCDA models. In [13], the authors used an approach based on the re-identification of criteria weights using stochastic methods such as Genetic Algorithm (GA), Differential Evolution (DE), and Particle Swarm Optimization (PSO). On the other hand, the study [11] presented the possibility of re-identifying the MCDA model using the Stochastic IdenTifiCation Of Models (SITCOM) approach, which determined preference values for characteristic objects. The SITCOM method was further developed into the Dynamic SITCOM (D-SITCOM) approach [12], which additionally considered the search for characteristic values when creating characteristic objects. In addition, machine learning models such as MultiLayer Perceptron (MLP) [14] are also used for the re-identification process. However, no research has been conducted on the re-identification process

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using the expected solution point, which reflects the desired outcome gathered from decision-makers. Notably, the ESP concept has demonstrated high efficacy in addressing multi-criteria problems through its personalized assessment approach [21]. Consequently, the recognition of this information gap served as the primary motivation for undertaking the present study.

This paper combines a stochastic method named Particle Swarm Optimization (PSO) to search for a single reference point determined as ESP defined in the SPOTIS method. This approach becomes an alternative to the ISP-SPOTIS method, where difficulties are encountered in re-identifying the decision-makers preferences due to how the ISP is determined based on the boundaries of the decision problem. By combining the PSO and ESP-SPOTIS methods, it is possible to direct the re-identification process toward personalized decision-making, increasing the effectiveness of the decision models' determination. Moreover, the novelty of our study is the demonstration of the possibility of updating an already re-identified model, which does not apply to the previously introduced approaches. The work's main contribution is the possibility of finding an expectation point to create an analogous reference model. Moreover, the proposed approach provides additional information as an ESP that can help interpret the decision-maker's preferences.

The paper is organized as follows. The 2 Section presents a literature review on how decision-makers convey their preferences to MCDA methods. The 3 Section presents preliminaries of the SPOTIS approach used and the correlation coefficients of the rankings. The 4 Section presents a proposed approach for re-identifying MCDA models called SESP-SPOTIS and research on this approach. The 5 Section presents conclusions and future research directions.

## 2 Literature review

Increasingly, research on Multi-Criteria Decision Analysis (MCDA) methods has focused on approaches related to processing decision-makers' preferences. In practice, the most commonly used technique is to assign weights to criteria that add up to unity. However, arbitrarily determining these values by decision-makers can be problematic. Consequently, criteria comparison methods are used to process their knowledge. The classic technique is the Analytic Hierarchy Process (AHP) method, which compares criteria using Saaty's scale. However, this approach is fraught with the paradox of reverse rankings, which can make its application unstable with constantly changing sets of alternatives.

However, the technique of comparing criteria is being developed in new approaches. The Best Worst Method (BWM) [18] focuses on comparing the best (best) and worst (worst) criteria in the context of the Multi-Criteria Decision Analysis problem under consideration. A similar approach is used in the Full Consistency Method (FUCOM) [15], which uses linear programming to determine criterion weights. However, these methods of determining weights only partially solve the problem of communicating the decision-maker's preferences.

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Preference function modeling is an alternative method for conveying a decision-maker's preferences. One of the classic approaches that focuses on this methodology is the Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) [1]. It allows the use of a variety of preference functions that can better reflect the specific preferences of the decision-maker. The PROMETHEE method evaluates alternatives based on the decision-maker's predefined preference functions, which allows for a more nuanced analysis than a simple assignment of weights. However, as in the case of transferring preferences through weights, the use of preference functions also does not guarantee that the MCDA technique is entirely immune to reverse rankings of alternatives.

Given the ability of decision makers to express preferences, the problem of re-identifying Multi-Criteria Decision Analysis (MCDA) models arises. There are many approaches to conveying preferences by the decision maker, such as using weights, pairwise comparisons, preference functions, or characteristic scores. This paper will focus on one aspect of re-identifying MCDA models, specifically the search for a reference point.

Table 1: Overview of MCDA methods based on reference points.

| **Name** | **Acronym** | **Reference point** | **Ref.** |
|---|---|---|---|
| Evaluation based on Distance from Average Solution | EDAS | Average solution for each criterion ($AV$) | [8] |
| Measurement of Alternatives and Ranking according to COmpromise Solution | MARCOS | Ideal solution ($AI$)<br>Anti-ideal solutuion ($AAI$) | [23] |
| COmbinative Distance-based ASsessment | CODAS | Negative Ideal Solution ($NS$) | [9] |
| Stable Preference Ordering Towards Ideal Solution | SPOTIS | Ideal Solution Point ($ISP$)<br>Expected Solution Point ($ESP$) | [4] |
| Technique for Order of Preference by Similarity to Ideal Solution | TOPSIS | Positive Ideal Solution ($PIS$)<br>Neagative Ideal Solution ($NIS$) | [6] |
| VIekriterijumsko KOmpromisno Rangiranje | VIKOR | Maximum criteria values ($f^*$)<br>Minimum criteria values ($f^-$) | [5] |
| Characteristic Objects METhod | COMET | Characteristic objects ($CO$) | [19] |
| Preference Ranking On the Basis of Ideal-average Distance | PROBID | Average solution ($\overline{A}$)<br>Positive ideal solutions ($PISs$)<br>Negative ideal solutions ($NISs$) | [26] |
| Compromise Ranking of Alternatives from Distance to Ideal Solution | CRADIS | Ideal Solution ($TI$)<br>Anti-ideal Solution ($TAI$) | [16] |
| Election based on Relative Value Distances | ERVD | Reference points ($\mu$)<br>Positive ideal solution ($PIS$)<br>Negative ideal solution ($NIS$) | [22] |
| Reference Ideal Method | RIM | Reference ideal ($s_j$) | [2] |

More recent research has focused on exploring the possibilities offered by the decision maker's transfer of one or more reference points aimed at optimizing and better adjusting the decision model. Introducing a reference point enables the implementation of nonlinear preference modeling, which is a significant step forward in personalizing the decision-making process. Among the most prominent methods using reference points are the approaches already mentioned, such as Reference Ideal Method (RIM), Stable Preference Ordering Towards Ideal

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Solution (SPOTIS), and Characteristics Objects Method (COMET). In addition, it is worth noting that there is a wide range of methods based on reference points, as illustrated by the Table 1.

## 3 Preliminaries

### 3.1 SPOTIS

The SPOTIS method, which stands for Stable Preference Ordering Towards Ideal Solution, differs from other MCDA approaches by incorporating the notion of reference objects. While methods like TOPSIS and VIKOR establish these objects based on a decision matrix, SPOTIS requires explicitly defined data boundaries. By employing this strategy to outline the domain of the decision problem, it becomes feasible to stabilize the ranking of alternatives towards the Ideal Solution Point (ISP), thus mitigating the occurrence of the Rank Reversal (RR) paradox. Typically, the ISP is determined based on the values associated with each criterion type (e.g., cost or profit). Establishing data boundaries is a crucial step in the initial phase of applying this method. For each criterion $C_j$, it's essential to select the maximum $S_j^{max}$ and minimum $S_j^{min}$ bounds. The Ideal Solution Point $S_j^*$ is defined as $S_j^* = S_j^{max}$ for profit criteria and $S^*j = Sj^{min}$ for cost criteria.

Additionally, as illustrated in [4], the SPOTIS approach allows for the utilization of any Expected Solution Point (ESP) in place of ISP. When employed, ESP generates a ranking specific to the subjectively chosen solution, proving beneficial when decision-makers seek a solution tailored precisely to a particular problem rather than a general ideal solution within the problem domain. The Expected Solution Point values $S_j^*$ should be chosen within the defined bounds of the decision problem $[S_j^{min}, S_j^{max}]$. Subsequently, the ESP vector $S^*$ should replace ISP in Equation (2) during the SPOTIS calculation procedure [21]. Below, the subsequent steps of the SPOTIS method are outlined.

**Step 1.** Definition of decision matrix.

The decision matrix describes the characteristics of considered alternatives under selected criteria. The formal notation of the decision matrix could be defined as (1):

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{im} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & x_{nm} \end{bmatrix} \tag{1}$$

where $x_{ij}$ is the attribute value of the $i$-th alternative for $j$-th criterion.

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**Step 2.** Calculation of the normalized distances from ISP (2):

$$d(A_i, S_j^*) = \frac{|S_{ij} - S_j^*|}{|S_j^{max} - S_j^{min}|} \tag{2}$$

**Step 3.** Calculation of weighted normalized distances $d(A_i, S^*) \in [0, 1]$ as (3):

$$d(A_i, S^*) = \sum_{j=1}^{N} w_j d_{ij}(A_i, S_j^*) \tag{3}$$

**Step 4.** Ranking calculation.

The final ranking of alternatives should be determined based on the value of $d(A_i, S^*)$. Better evaluated decision variants have smaller values of $d(A_i, S^*)$, thus should be placed higher in the ranking.

### 3.2 Weighted Spearman's correlation coefficient

The Weighted Spearman's correlation coefficient ($r_W$), proposed by [3], extends the traditional Spearman coefficient by integrating weights. It computes the correlation between two rankings, both of size $N$, where $x_i$ represents the position in the first ranking and $y_i$ indicates the position in the second ranking (4).

$$r_W = 1 - \frac{6 \cdot \sum (x_i - y_i)^2 ((n - x_i + 1) + (n - y_i + 1))}{n \cdot (n^3 + n^2 - n - 1)} \tag{4}$$

### 3.3 WS rank similarity coefficient

The Weighted Similarity ($WS$), proposed by [20], presents itself as an asymmetric measure of ranking similarity. In contrast to conventional methods, it places particular emphasis on alterations occurring at the top of rankings. Consequently, the correlation undergoes a significant decrease if, for example, there is an interchange between the first and last positions. The $WS$ rank similarity coefficient is confined within the interval [0, 1], where zero indicates uncorrelated rankings, while a value of one signifies identical rankings. Computed for two rankings, $x_i$ and $y_i$, both with a size of $N$, the similarity value is determined as (5):

$$WS = 1 - \sum \left( 2^{-x_i} \frac{|x_i - y_i|}{\max |x_i - 1|, |x_i - N|} \right) \tag{5}$$

## 4 The proposed approach

This section will present a proposed approach called Stochastic Expected Solution Point SPOTIS (SESP-SPOTIS). This approach aims to identify the expected solution point using a stochastic method. It incorporates both the simplicity of the SPOTIS method, as discussed in the Section 3.1, and the re-identification capabilities of the decision model. In the context of possible nonlinearity associated with selecting the expected solution point, the re-identification

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approach becomes the answer to the problem of identifying nonlinear decision models. In the case of the present work, the stochastic technique used for this purpose is Particle Swarm Optimization (PSO). The main steps of this approach can be presented as follows:

**Step 1. Select a dataset.** The dataset should include the decision matrix of the given decision problem. Additionally, it should contain information such as weights criteria vectors ($W$), a criteria types vector ($T$), and a ranking vector ($R$).

**Step 2. Select a stochastic optimization method.** In this step, choose a stochastic method for solving the optimization problem and select its parameters. In this paper, Particle Swarm Optimization (PSO) was selected as the stochastic optimization method. PSO is a popular technique for stochastic optimization problems and allows for modeling flexible objective functions [27]. Its algorithm is presented in Algorithm 1.

**Algorithm 1** Particle Swarm Optimization (PSO)

```
Initialize X, V, P, P_value, G, G_value
for iteration ← 1 to max_iterations do
    for each particle i do
        Update velocity and position
        Clip position to within bounds
        Evaluate objective function
        if f_i > P_value[i] then
            Update personal best
        end if
    end for
    Update global best
    if convergence_criteria_met() then
        break
    end if
end for
Output: G, G_value
```

**Step 3. Model training.** Training the model is done using the stochastic optimization algorithm and the fitness function, which can be determined as presented in Algorithm 2.

**Algorithm 2** Fitness Function

```
procedure FITNESS(solutions):
    base.esp ← solutions
    preference ← base(C, solutions, T)
    return rw(base.rank(preference), R)
end procedure
```

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### 4.1 Re-identification - exemplary study case

This paper presents a simple example of expert model re-identification in the context of multi-criteria decision analysis. Suppose a particular set of evaluated samples is evaluated by a multi-criteria decision analysis model. However, the multi-criteria decision analysis model itself and its parameters are unknown. In this case, when wanting to re-identify such a model, the SPOTIS method can be used. The SPOTIS method has two possible modeling routes. The first is to use an ISP point created based on the values derived from the model boundaries for each criterion. In such a model, the only possible representation of the decision-maker's preferences is expressed in terms of weights. The second way of modeling is to use the ESP point, which the decision-maker chooses. This point determines the most preferred alternative that we would like to obtain. Therefore, by taking the second route, it is possible to find an ESP point that can produce a similar model to the reference one.

Assume that there is a multi-criteria evaluation problem, where a decision matrix containing attribute values for 10 alternatives against two criteria is available. Suppose the evaluation model, which has not yet been applied, runs from 0 (smallest cutoff value) to 1 (largest cutoff value) for each criterion. In order to find an Expected Solution Point that will help us create a similar evaluation model, a PSO method will be used. This method is widely used for optimization problems and involves simulating the behavior of a swarm of particles in the solution search space to find the optimal point. To implement the PSO algorithm, an implementation of the MealPy library will be used, which provides ready-made tools for solving optimization problems. The version of the MealPy library used in this implementation of the library is 2.5.4 [25]. For this particular problem at hand, it is necessary to define a fitness function that will evaluate the quality of solutions in the context of ESP search. A detailed description of the PSO method and the fitness function can be found above. It is also worth mentioning the selected parameters for the PSO algorithm, such as the number of particles ($pop_{size} = 20$), weight coefficients ($c_1 = c_2 = 2.05$), maximum number of iterations ($epoch = 1000$), which have been adjusted for our specific problem in order to achieve optimal results.

Using Figure 1, the unidentified reference model and the SEPS-SPOTIS derived model are shown. It can be seen that in the case of the re-identification of such a model, the distribution of preferences was similarly mapped. In the case of the present re-identification, the rankings of the alternatives are very close to each other, where a Weighted Spearman correlation coefficient value of 0.96915 and a WS coefficient value of 0.94152 were obtained. On the other hand, referring to the distance of the found ESP from the extremum of the present unknown model, it is 0.20341 . Such re-identification makes it possible to obtain similar evaluations to the reference evaluations, by which it is also possible to evaluate a new set of alternatives.

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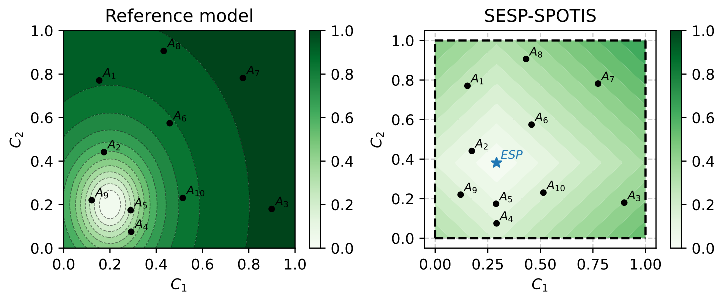


Fig. 1: Reference model and model re-identified using the SESP-SPOTIS approach.

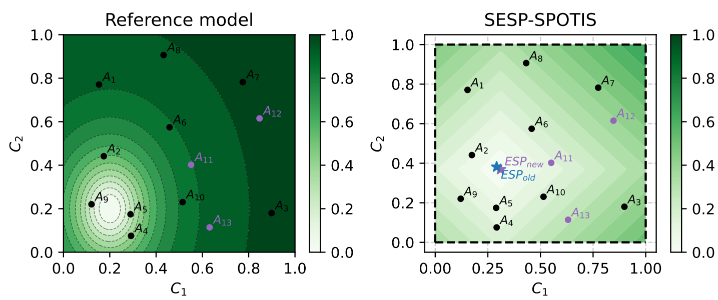


Fig. 2: Reference model and updated SEPS-SPOTIS model with 3 additional alternatives.

In the case of re-identification, it is also possible to update the SESP using the newly evaluated samples. Suppose that in addition to the set on which working also obtained additional evaluated alternatives. In this case, another re-identification of the reference model is possible. The SEPS retrieved in the previous re-identification is set as the search's starting point. Then the whole process looks identical, however, in the case of this training, the training set is a set of 13 alternatives, not 10 alternatives. The Figure 2 shows the result of the re-identification of the model. The shift of the retrieved ESP relative to the old ESP can be seen. The distance between the two is 0.02232. In addition, the distance between the extremum of the unknown reference model and the retrieved new ESP has been reduced and is 0.20159. This means that re-identifying the model using the SESP-SPOTIS approach makes it possible to increase the accuracy.

Using Figure 3, a comparison of the re-identified models on a test set of 10 alternatives is presented. Figure 3a shows the comparison of the re-identified

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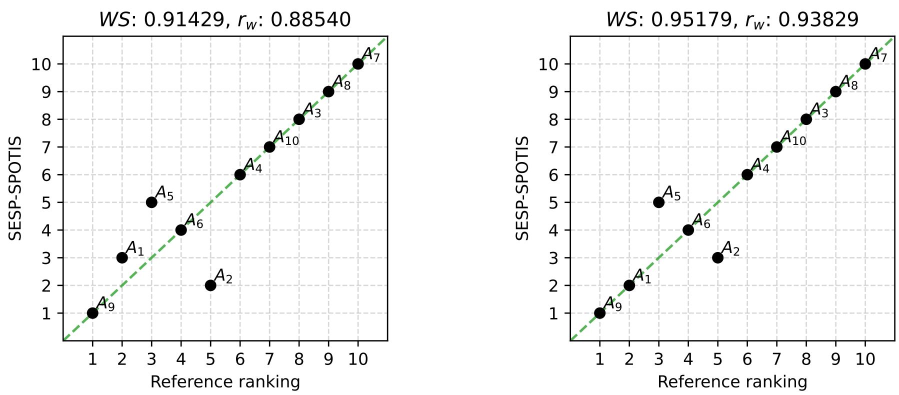
model on the 10 test alternatives with the reference model, while Figure 3b shows the comparison of the updated re-identified model by 3 test alternatives with the reference model. It can be observed that for the updated re-identified model with 3 alternatives, the correlation is higher with the ranking obtained with the reference model than for the re-identified model with 10 alternatives. For the model updated with 3 alternatives, the correlation of the ranking with the reference model was based on the index $r_w$, a value of 0.93829, while using the coefficient $WS$, a value of 0.95179. Referring to the re-identified model on 10 alternatives without updating, its ranking correlation with the reference model is $r_w = 0.88540$ and $WS = 0.91429$.




(a) Re-identification on 10 alternatives.

(b) Re-identification update with 3 alternatives.

Fig. 3: Comparison on a test set of rankings from the unknown and re-identified model (SESP-SPOTIS).

### 4.2 Effectiveness

This section will focus on a simulation study related to testing the proposed SESP-SPOTIS approach. The unknown MCDA model shown in the previous section is the reference model in this section. Figure 4 shows a two-dimensional histogram through which the distribution of ESP values for the two considered criteria can seen. This distribution was obtained from simulations for 1000 random training sets of size 10 alternatives. However, it can observed that the values of searched ESPs concentrated around the extremum of the unknown MCDA model. Extremes are often searched for due to the distribution of alternatives existing in the training set. The mean values of ESPs that were searched are for the criterion $ESP_{C_1}^{Mean} = 0.19164$ and the criterion $ESP_{C_2}^{Mean} = 0.30834$. The standard deviation among values for criterion $ESP_{C_1}^{STD} = 0.11450$ and criterion $ESP_{C_2}^{STD} = 0.13275$.

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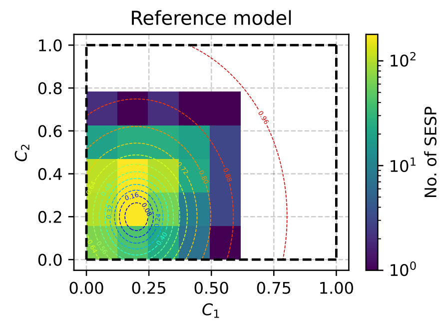


Fig. 4: Two-dimensional histogram of ESP values for the two considered criteria.

The SESP-SPOTIS approach was also compared with the ISP-SPOTIS approach, which uses an ideal point formed from the model's outliers. The Figure 5 shows the distributions of the values of correlation coefficients for comparisons of rankings derived from the reference model and the SESP-SPOTIS and ISP-SPOTIS models. A significant difference in the accuracy of the approaches can be observed. The SESP-SPOTIS model searches for the expected point based on the evaluated alternatives, and its accuracy is much higher than that of the ISP-SPOTIS model, which is based only on the model boundaries. The average values of $WS$ and $r_w$ obtained by the SESP-SPOTIS model are 0.95492 and 0.94101, while the average values of $WS$ and $r_w$ obtained by the ISP-SPOTIS model are 0.46773 and -0.24206, respectively.

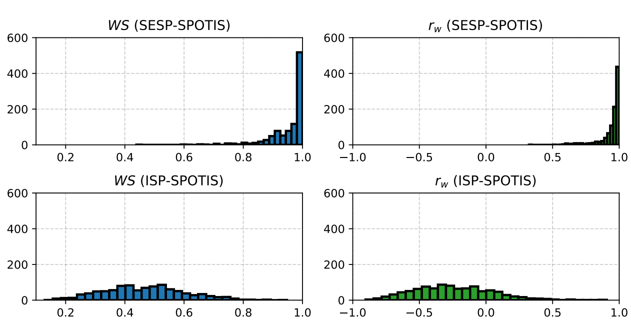


Fig. 5: Distributions of obtained values of correlation coefficients from comparisons of rankings with the reference model and SPOTIS models.

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**Algorithm 3** Demonstration of the experimental procedure.

```
Input: N ← 1000
Input: criteria ← 2
for num_alt in [5, 10, 25, 50, 100] do
    coefficients_train,coefficients_test ← Coefficients()
    for i = 1 to N do
        1) Train results:
        alternatives_train ← generate_alternatives(num_alt, criteria)
        weights ← equal_weights(alternatives_train)
        reference_train ← ref(alternatives_train)
        model ← sesp_spotis(alternatives_train, reference_train, weights)
        result ← model.pred(alternatives_train)
        coefficients_train.add(WS(result, reference_train))
        coefficients_train.add(rw(result, reference_train))
        2) Test results:
        alternatives_test ← generate_alternatives(10, criteria)
        result ← model.pred(alternatives_test)
        reference_test ← ref(alternatives_test)
        coefficients_test.add(WS(result, reference_test))
        coefficients_test.add(rw(result, reference_test))
    end for
    save_coefficients(coefficients_train)
    save_coefficients(coefficients_test)
end for
```

Simulation studies were also performed for 1000 random sets of alternatives, where the number of alternatives $5, 10, 25, 50, 100$ was taken as the training set, while the test set of alternatives had a fixed size of 10 alternatives. The possibility of re-identifying the reference model was tested using the SESP-SPOTIS approach, where the comparison of the accuracy of the re-identified model with the reference model was verified using the correlation coefficients of WS and rw rankings. The Algorithm 3 shows the procedure of the conducted study.

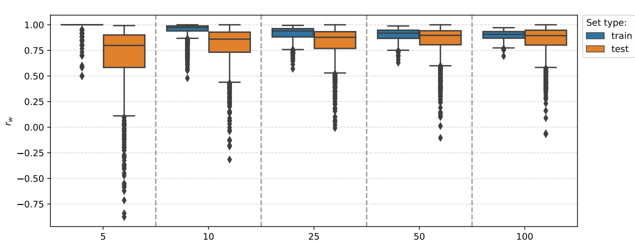


Fig. 6: Distribution of weighted Spearman rank correlation coefficient values ($r_w$) for reference model comparisons and SESP-SPOTIS for training and test sets.

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Figure 6 displays results from the reference and re-identified models using the SESP-SPOTIS approach, represented by the weighted Spearman coefficient ($r_w$). Across test datasets, the average $r_w$ was 0.689 to 0.842, and for training datasets, it was 0.899 to 0.978. Standard deviation varied from 0.161 to 0.309 for tests and 0.044 to 0.076 for training, with smaller datasets showing higher variability. The $r_w$ range was -0.877 to -0.069 (test) and 0.479 to 1.000 (training). The analysis indicates strong correlation between models and reference, but smaller datasets had increased variability and reduced correlation.

Figure 7 presents results from the reference and re-identified models using the SESP-SPOTIS approach, shown by the $WS$ correlation coefficient. For test sets, the mean $WS$ ranged from 0.789 to 0.880, and for training sets, from 0.905 to 0.982. Standard deviation was between 0.084 to 0.163 for tests and 0.048 to 0.054 for training, with higher variability in smaller sets. Minimum $WS$ values were 0.137 to 0.461 (test) and 0.643 to 0.672 (training), and maximum values were 0.884 to 1.000 (test) and 0.986 to 1.000 (training). The re-identified models showed high correlation with the reference, but smaller datasets had increased variability and decreased correlation.

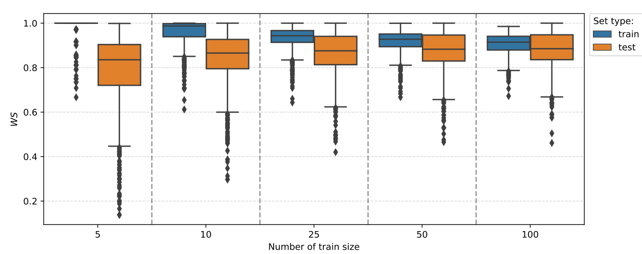


Fig. 7: Distribution of weighted rank correlation coefficient values ($WS$) for reference model comparisons and SESP-SPOTIS for training and test sets.

## 5 Conclusions

This paper introduces a novel re-identification approach for Multi-Criteria Decision Analysis (MCDA) models using the stochastic Expected Solution Point search method in SPOTIS. It explores updating re-identified models with new evaluations and evaluates accuracy through training and test sets using rank correlation coefficients $r_w$ and $WS$. Results show the SESP-SPOTIS method identifies models close to the reference and provides valuable expected solution points for interpreting decision maker preferences.

Future research should consider developing this approach, taking into account uncertainty based on fuzzy sets and their generalizations. Other stochastic search methods should also be explored in the context of re-identifying MCDA models. In addition, it is worth considering the possibility of searching for multiple points of the expected solution under ESP.

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