Enhancing the realism of wildfire simulation using Composite Bézier curves

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Abstract. One of the consequences of climate change is the increase in forest fires around the world. In order to act quickly when this type of natural disaster occurs, it is important to have simulation tools that allow a better approximation of the evolution of the fire, especially in Wildland Urban Interface (WUI) areas. Most forest fire propagation simulators tend to represent the perimeter of the fire in a polygonal way, which often does not allow us to capture the real evolution of the fire in complex environments, both at the terrain and vegetation levels. In this work, we focus on Elliptical Wave Propagation (*EWP*) based simulators, which represent the perimeter of the fire with a set of points connected to each other by straight lines. When the perimeter grows and new points must be added, the interpolation method used is linear interpolation. This system generates unrealistic shapes of fires. In this work, an interpolation method leveraging *Composite Bézier Curves* (*CBC*) is proposed to generate fire evolution shapes in a more realistic way. The proposed method has been incorporated into FARSITE, a well-known *EWP*-based forest fire spread simulator. Both interpolation methods have been applied to ideal scenarios and a real case. The results show that the proposed interpolation method (*CBC*) is capable of generating more realistic fire shapes and, in addition, enables the simulator the ability to better simulate the spread of fire in WUI zones.

Keywords: Interpolation · Forest Fire perimeter · Bezier curve.

1 Introduction

The role of computational science in addressing environmental challenges such as wildfires, is increasingly recognised as part of the broader pursuit of sustainability. Wildfires are increasingly recognised as a major environmental and societal challenge. The frequency and intensity of these events have risen notably in recent years, leading to significant ecological, economic, and social impacts. Climate change, marked by rising temperatures and changing precipitation patterns, has exacerbated the conditions that lead to wildfires [10, 8, 13]. Effective

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wildfire management, facilitated by advanced simulation tools, contributes significantly to the safety and well-being of societies, as well as to the protection and conservation of natural ecosystems. The accuracy and realism of fire behaviour simulations are crucial for this effective wildfire management. Traditional simulation methods, while useful, often lack the fine scale detail necessary to capture the complex nature of wildfire spread. This gap in simulation fidelity can lead to challenges in predicting fire behaviour, especially in heterogeneous landscapes with variable fuel and weather conditions.

In response to this challenge, numerous mathematical models and simulators have been developed in the last decades [6, 7], which are broadly categorized based on their spread strategy into three types: *Cellular Automata (CA)*, *Elliptical Wave Propagation (EWP)* and *Level Set Method (LSM)*. Fire spread in the *CA* models is performed based on a grid of cells, where the state of each cell could be either burned or unburned [1]. *EWP* models treat the fire perimeter as a discretized curve (set of points) offering detailed and dynamic representations of fire spread [2]. Finally, *LSM* employs the Hamilton-Jacobi equation to define the fire front implicitly through a level-set function [4, 11].

Focusing on the realism of the simulations provided by the simulators that use *CA* and *EWP* as fire front propagation strategies, a relevant issue can be found, the polygonal shape of the results. Both, *CA* and *EWP* methods tend to generate fire perimeters with polygonal shapes instead of curved shapes. This issue is not so notable in *LSM* since this approach does not discretize the fire front. The reason for this behavior in the case of *CA*-based simulators, is the use of cells as a propagation unit, while in *EWP*-based models, the main problem lies in the interpolation method used. As the forest fire evolves, new points must be added to the representation of the fire perimeter to keep the resolution of the simulation limited by a predetermined value. This point addition is done through linear interpolation, which generates straight shapes instead of smooth curves.

The main objective of this work is to emulate the dynamic and curved characteristics inherent in the spread of a forest fire by applying *Composite Bézier Curves*. This concept has been widely applied in the area of computing graphics but, in this paper, we are transferring its applicability to a completely different research field. The proposed methodology uses an interpolation technique that strategically introduces points on the front obtained from the composition of the generated curves. The resulting addition of points gracefully articulates a more realistic depiction of the fire's boundary. What distinguishes this approach from traditional polygonal methods is that it allows to capture the authentic, non-polygonal behavior exhibited by a wildfire. Furthermore, the proposed methodology does not imply extra computing time since it has been designed to have the same complexity as current linear interpolations ($O(n)$). In order to analyze the behaviour of this proposal when simulating the behaviour of real forest fires in complex scenarios, we have used FARSITE as a simulation framework. FARSITE has been chosen for being the most widely used forest fire simulator that incorporates the *EWP* spread method. The proposed interpolation method

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has been codified in FARSITE changing its linear interpolation method to the proposed one but keeping the rest of the code intact. FARSITE has been used to simulate the evolution of a real wildfire using both interpolation methods separately. The results show that the proposed method not only provides more realistic forest fire perimeters, but also enables the simulator the ability to spread the fire through areas that would not otherwise be reached.

The rest of the paper is organized as follows. In section 2 a basic description of FARSITE is introduced. Section 3 includes the description of the proposed *CBC* interpolation method. The experimental study is reported in section 4 and, finally, section 5 summarises the main conclusions of this work.

2 Forest Fire Spread Modelling

Modelling and understanding the behaviour of wildfire is a complex process that involves a lot of different fields (physics, forestry, chemistry, etc). However, with in special conditions (flat terrain, no wind and homogeneous fuel), the propagation of the fire front can be simplified by a circle [12]. Other well known special (ideal) cases are those with either constant slope or, flat terrain with constant wind speed and direction. Assuming these conditions, the propagation of the fire front describes an ellipse [3]. This behaviour is shown in Figure 1 where the fire evolution in controlled laboratory experiments are depicted [12].

Fig. 1: Elliptical behaviour observed in laboratory experiments [12].

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FARSITE is a widely employed simulator that adopts *Elliptical Wave Propagation* as its primary method for modelling fire spread across landscapes. Combining this propagation scheme with the Huygens' principle, the fundamental framework of FARSITE's methodology is obtained. FARSITE works iteratively by modifying the location of the fire front in time steps of preset duration. In each iteration, FARSITE dynamically updates the fire front by strategically placing points along elliptical waves. This adaptability allows the simulator to navigate diverse landscapes, capturing the nuanced path and intensity of the spreading fire. The Huygens' principle, a cornerstone of FARSITE's approach, involves points along the elliptical wavefront acting as sources of secondary waves. This collective influence shapes the evolving wavefront, enhancing the precision of fire spread predictions. Figure 2 shows a basic scheme of how the *EWP* method is used to spread an initial fire front to an evolved fire perimeter.

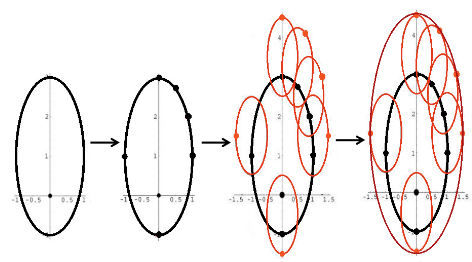

Fig. 2: Basic scheme of the *Elliptical Wave Propagation* (*EWP*) approach.

However, despite its robust iterative dynamics, FARSITE encounters challenges related to the *EWP*. As the fire progresses, the gradual drift of points necessitate careful consideration. Maintaining simulation accuracy and resolution becomes crucial, additionally, the autonomy in the spread of each point contributes to a lack of knowledge regarding previously burned areas. The decentralized nature of point propagation creates a potential gap in understanding the fire's history, necessitating innovative solutions to overcome this information limitation and enhance the overall accuracy of the simulation.

In facing the lack of knowledge regarding previously burned area, FARSITE introduces the usage of a normal vector to the perimeter, which is crucial in guiding the fire spread. The normal vector of a given point is computed based on its surrounding points and aligned with the existing momentum. Figure 3 illustrates schematically how the normal vectors for two different points (grey and red coloured) are obtained from its neighbouring points. To evaluate the direction of the normal vector for both points, the perpendicular direction of the segment that joins the corresponding two neighbouring points is used. Therefore, the location of the two neighbours of a given point have a direct impact in the normal vector direction. Later we will return to this characteristic since, as we will see, it is a relevant point in the proposal of this work.

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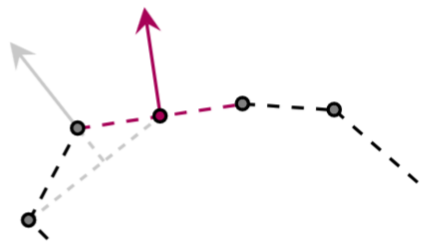

Fig. 3: Normal vector for two points (grey and red) when interpolation has been done linearly.

To address spatial accuracy, FARSITE introduces a rediscretization process to counteract the gradual point density drift. FARSITE incorporates the `perimeter_resolution` (PR) parameter that defines the maximum distance between two consecutive points within the fire front, determining when a new point should be added to maintain simulation resolution limited by the PR value. The method employed by FARSITE to introduce new points along the elliptical wavefront during the rediscretization stage is linear interpolation. This technique adds a new perimeter point at the midpoint along the segment connecting two existing points on the wavefront when required. While this rediscretization strategy effectively addresses spatial accuracy concerns, it introduces a challenge related to the Huygens' principle. As new points are inserted within the area enclosed by the elliptical wavefront, the smoothness or curvature of the wavefront is disrupted. This disruption not only impacts the smoothness of the wavefront but also has implications for the computation of the normal vector for the neighbour points, so this disruption propagates throughout the iterations. Later on, in the section devoted to explain the experimental study carried out in this work, a deep analysis of this issue is done.

3 Methodology

In this section, a detailed exploration of the mathematical intricacies behind *Composite Bézier Curves* (*CBC* for short) is introduced. The main emphasis lies not only in understanding their application but also in presenting an efficient computational approach. The proposed *CBC* method exhibits a complexity of $O(n)$, ensuring that the computational time aligns with the basic linear interpolation akin to what FARSITE uses. This strategic approach allows us to maintain computational efficiency while enhancing the simulation's fidelity in capturing the nuanced behavior of a forest fire through *Bézier* curves.

3.1 Composite Bézier Curves

The foundation of the *CBC* method lies in the composition of Bézier curves. The Bézier curves under consideration are cubic Bézier curves, defined based on four points, as it is illustrated in Figure 4. Two of these points (P_0 and P_1) act

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as the starting and ending points of the curve, while the other two (σ_0 and ρ_0) serve as anchor points that influence the curve's shape and directionality.

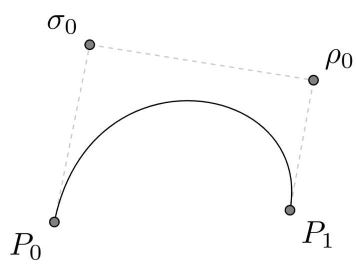

Fig. 4: Representation of a Cubic Bézier Curve.

The Cubic Bézier curve (B) depicted in Figure 4 has the following generic formula:

$$B(t) = (1-t)^3 \cdot P_0 + 3t(1-t)^2 \cdot \sigma_0 + 3t^2(1-t) \cdot \rho_0 + t^3 \cdot P_1 \\ 0 \leq t \leq 1 \in \mathbf{R} \quad (1)$$

In the context of *Composite Bézier Curves*, each curve is defined by a pair of points from the fire perimeter — marking the start and end — coupled with two anchor points yet to be determined. The task at hand involves determining the anchor points, a process we will elucidate later in this section. With as many curves as there are points on the fire perimeter, the general formulation for each Bézier curve (Γ_i) describing the perimeter is expressed through Equation (2).

$$\Gamma_i(t) = (1-t)^3 \cdot P_i + 3t(1-t)^2 \cdot \sigma_i + 3t^2(1-t) \cdot \rho_i + t^3 \cdot P_{i+1} \\ i \in \{0, \ldots, n-1\},\ 0 \leq t \leq 1 \in \mathbf{R} \quad (2)$$

In Equation (2), σ_i and ρ_i represent the anchor points to be determined, P_i denotes the i-th point on the perimeter, P_{i+1} is the following point clockwise for all curves, and n is the number of points in the perimeter. As the perimeter forms a closed shape, the point following the last perimeter point seamlessly connects to the first point of the perimeter, so P_n is P_0.

The challenge is to find the σ_i and ρ_i for each curve in a way that leaves us with integrated curves and smooth composition. As we currently have $2n$ unknowns factors, we need to produce $2n$ equations to obtain a compatible determined system. Furthermore, since we want the composition to be differentiable for smoothness, we propose it to be twice differentiable to obtain the $2n$ equations. This differentiability condition is expressed in Equations (3) and (4).

$$\Gamma_i^{'}(1) = \Gamma_{i+1}^{'}(0),\ \ i \in \{0, \ldots, n-1\} \quad (3)$$

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$$\Gamma_i^{''}(1) = \Gamma_{i+1}^{''}(0), \quad i \in \{0, \ldots, n-1\} \tag{4}$$

In Equations (3) and (4), it should be noted that since we are working with a circular system, Γ_n is Γ_0, as it will happen elsewhere in our methodology.

Using the definition of $\Gamma_i(t)$ from Equation (2), we can calculate its first and second derivatives. By using these definitions and Equations (3) and (4), we can derive Equations (5) and (6).

$$\Gamma_i^{'}(1) = \Gamma_{i+1}^{'}(0) \iff \sigma_{i+1} + \rho_i = 2P_{i+1} \tag{5}$$

$$\Gamma_i^{''}(1) = \Gamma_{i+1}^{''}(0) \iff \sigma_i + 2\sigma_{i+1} = 2\rho_i + \rho_{i+1} \tag{6}$$

Therefore, we obtain the system of equations in Equation (7) with $2n$ unknowns and $2n$ equations.

$$\begin{cases} \sigma_{i+1} + \rho_i = 2P_{i+1}, & i \in \{0, \ldots, n-1\} \\ \sigma_i + 2\sigma_{i+1} = 2\rho_i + \rho_{i+1}, & i \in \{0, \ldots, n-1\} \end{cases} \tag{7}$$

Applying the substitution method, we obtain a system containing only n unknowns, which are the σ_i with $i \in \{0, \ldots, n-1\}$, and with n equations.

$$\{\sigma_{i-1} + 4\sigma i + \sigma_{i+1} = 2(2P_i + P_{i+1}), \quad i \in \{0, \ldots, n-1\} \tag{8}$$

It is essential to note that this system is a circular tridiagonal system, and for this reason, it can be expressed in the following generic form of Equation (9). We express it in this generic form because the resolution methodology presented below is expressed using this form, as it can be used for all circular tridiagonal systems.

$$\{b_i \cdot \sigma_{i-1} + a_i \cdot \sigma i + c_i \cdot \sigma_{i+1} = r_i, \quad i \in \{0, \ldots, n-1\} \tag{9}$$

Based on the methodology of [9], which uses Gaussian elimination and a iterative reparametrization, we obtain an equivalent system. The reparametrization used starts with the initial parameters in Equation (10),

$$\begin{cases} \alpha_0 = \dfrac{-1}{a_0} \\ \beta_0 = b_1\alpha_0 \\ \delta_0 = b_0 \\ \epsilon_0 = -c_{n-1}\alpha_0 \\ y_0 = r_0 \end{cases} \tag{10}$$

Then, we compute the iterative parameters, based on the previous ones, in Equation (11).

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$$\begin{cases} \alpha_i = \dfrac{-1}{a_i + \beta_{i-1} c_{i-1}} \\ \beta_i = b_{i+1} \alpha_i \\ \delta_i = \beta_{i-1} \delta_{i-1} & i \in \{1, \dots, n-2\} \\ \epsilon_i = \epsilon_{i-1} c_{i-1} \alpha_i \\ y_i = r_i + y_{i-1} \beta_{i-1} \end{cases} \tag{11}$$

Finally, we calculate the final parameters of Equation (12).

$$\begin{cases} \alpha_{n-1} = a_{n-1} + \beta_{n-2}(\gamma_{n-2} + c_{n-2}) - \epsilon_{n-2} c_{n-2} - \displaystyle\sum_{j=0}^{n-2} \epsilon_j \gamma_j \\ y_{n-1} = r_{n-1} + \beta_{n-2} y_{n-2} - \displaystyle\sum_{j=0}^{n-2} \epsilon_j y_j \end{cases} \tag{12}$$

Therefore, we obtain the following system of equations equivalent to Equation (9) using the described reparametrization:

$$\begin{cases} \sigma i + -c_i \alpha_i \cdot \sigma_{i+1} - \gamma_i \alpha_i \sigma_{n-1} = -y_i \alpha_i, & i \in \{0, \dots, n-3\} \\ \sigma_{n-2} - \alpha_{n-2}(c_{n-2} + \gamma_{n-2}) \sigma_{n-1} = -y_{n-2} \alpha_{n-2} \\ \alpha_{n-1} \sigma_{n-1} = y_{n-1} \end{cases} \tag{13}$$

Iterative starting from σ_{n-1}, we can solve the system, and the solution is given by Equation (14).

$$\begin{cases} \sigma_{n-1} = \dfrac{y_{n-1}}{\alpha_{n-1}} \\ \sigma_i = (c_i \sigma_{i+1} - y_i + \gamma_i \sigma_{n-1}) \alpha_i & i \in \{n-2, \dots, 0\} \end{cases} \tag{14}$$

Finally, using Equation (5), we also obtain the variables ρ_i. After solving the system and obtaining the variables σ_i and ρ_i, we have determined the curves that constitute the *Composite Bézier Curves*. These curves smoothly and cohesively describe and follow the fire perimeter, accurately capturing the dynamics and directionality of the fire spread.

To exemplify how this methodology works, a toy example is used. Figure 5a shows a set of points (grey points) that represent a certain fire perimeter just before the rediscretization process starts. The points have been represented by joining them with a line just to clarify the shape they form. Figure 5b depicts the two set of points that represent the fire front once the two interpolation methods have been applied. More precisely, the red points (red perimeter) correspond to the points obtained when using FARSITE's linear interpolation, whereas the green points (green perimeter) are the points obtained with the proposed *CBC*

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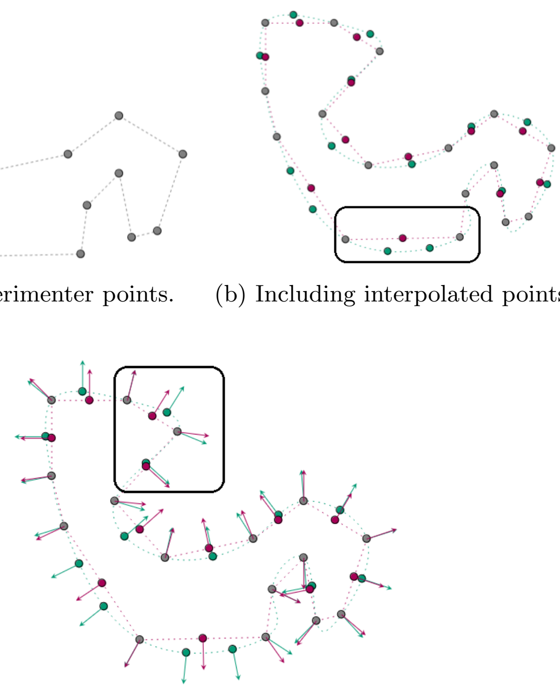

(a) Original perimenter points. (b) Including interpolated points.

(c) Normal vectors.

Fig. 5: Original perimeter points before any interpolation (grey) (a). Perimenter including interpolated points, the green ones corresponds to the interpolation done with the proposed method *CBC* and the red ones correspond to the points obtained applying the linear interpolation (b). Normal vectors of all perimeter points (c).

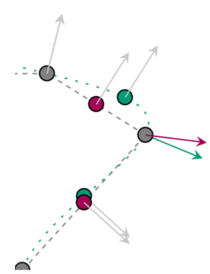

Fig. 6: Normal vectors for interpolated points (linear interpolation in red and *CBC* method in green) and original perimeter points before interpolation in grey

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interpolation method. As it can be observed, the red perimeter exhibits a polygonal shape, meanwhile the green perimeter draws a smoother curve shape. Furthermore, if we observe the framed area in Figure 5b in more detail, we can see that the interpolation process, in the case of the standard method, only introduces one point (red) between the two points of the original perimeter (grey points), while the *CBC* method adds two points (green). This example highlights that the interpolation used by FARSITE only adds one point in the center of the segment that joins the perimeter points between which it is detected that points need to be added. This process is carried out in this way even if the distance between points exceeds *PR*. However, the proposed *CBC* interpolation adds to the perimeter all required extra points to keep the perimeter resolution bounded by *PR*. Figure 5c shows the normal vectors for all points. Although at first glance it seems that the normal vectors for the two interpolation methods coincide, if we zoom in the framed area of Figure 5c (Figure 6), we can see that there is a slight difference between them. This difference is more pronounced in the grey points, that is, in the perimeter points before the rediscretization stage (interpolation process) starts. As it has been previously mentioned, since the two interpolation methods add points to the perimeter at different locations, the resulting neighbouring points for a given grey point could be very different. Figure 6 depicts this situation. In this figure, the two normal vector obtained when the interpolation method used is the linear one (red points) and the *CBC* method is applied (green point) are shown. As it can be observed, the obtained normal vectors, especially for the grey point, are quite different since the two neighbouring points in each case are in locations significantly different. This difference will propagate at each simulation step leading to relevant differences at the end of the complete simulations process. This issue will be later on analysed in this work, in the discussion of the real case.

4 Experimental study and Results

In this section, the experimental study carried out is reported. In order to analyze the behaviour of the proposed method, we have done two kind of experiments. On the one hand, ideal scenarios where most of the environmental conditions are quite controlled have been simulated to determine the effectiveness of the proposed interpolation method against the current basic approach. Being able to control variables such as wind and terrain allows the study to focus on the behavior of fire spread. On the other hand, a real wildfire scenario has been simulated to test the *CBC* methodology under complex scenarios. the application of these methods to a real wildfire event is crucial for assessing their practical utility and accuracy in replicating complex fire dynamics. This comparison aims to reveal not only how each method performs in theory but also their effectiveness in actual wildfire situations. In addition, the real wildfire event introduces complexities absent in ideal scenarios, presenting an opportunity to assess the practical utility of the *CBC* method under dynamic and heterogeneous conditions.

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4.1 Ideal Cases

To test the effectiveness of the proposed rediscretization method within the context of the FARSITE forest fire spread simulator, a set of experiments based on ideal cases have been designed. Ideal cases are those that the relevant environment conditions (slope, wind and vegetation) are controlled and constant. That is, for example, the basic ideal case consists of simulating the spread of a forest fire in a completely flat terrain, with homogeneous vegetation and no wind. Under this conditions, it is well known that the evolution of the fire develops in concentric circles [12, 5]. In particular, the results shown in this section corresponds to this ideal basic case, however, other ideal cases have been tested such as flat terrain and constant wind, terrain with a constant slope with and without constant wind and so on. The results of all these experiments exhibit similar behaviour to the basic one, therefore, we have chosen this case to compare the linear interpolation method used by FARSITE with running FARSITE including the proposed *CBC* interpolation method.

FARSITE exhibits the capability to maintain a circle-like fire shape when simulating the basic ideal case. However, as it has been previously mentioned, the current rediscretization method included in FARSITE adds new points at the midpoint between two neighbouring points when required. This method could locally alter the elliptical or circular shape of the fire perimeter turning it into a polygonal shape. To show such a behaviour, the ideal case has been simulated using, on the one hand, FARSITE with its standard linear interpolation method and, on the other hand, FARSITE has been executed by changing the linear interpolation to the proposed *CBC* interpolation method. Figure 7a shows the simulations results when using linear interpolations, meanwhile Figure 7b depicts the evolution of the fire when using *CBC* interpolation approach. The circular fire spread pattern, inherent to FARSITE's underlying principles, are faithfully reproduced by both methods. Although at first glance, the results obtained when simulating the basic ideal case seem identical, if we analyze both figures in more detail a subtle difference can be seen as it can be observed in Figure 7c. The *CBC* method introduces refinements, optimizing the representation of circular fire spread generating smoother perimeters. These results corroborate that the new interpolation methodology is capable of reproducing the circular evolution of the spread of a fire in an ideal scenario. Furthermore, it is capable of eliminating the polygonal appearance of the perimeters generated by FARSITE with its basic interpolation implementation.

4.2 Real Case

This section is devoted to study the results obtained in terms of realism of the fire perimeter, when applying both interpolation methods, the original scheme included in FARSITE and the proposal method (*CBC*) in a real wildfire. The study case corresponds to a forest fire that took place in *Pont de Vilomara* in Catalonia (north-east of Spain). This forest fire started on July 17, 2022, at 13:04 and it was completely controlled on July 18 at 00:20. It rapidly burnt around 14

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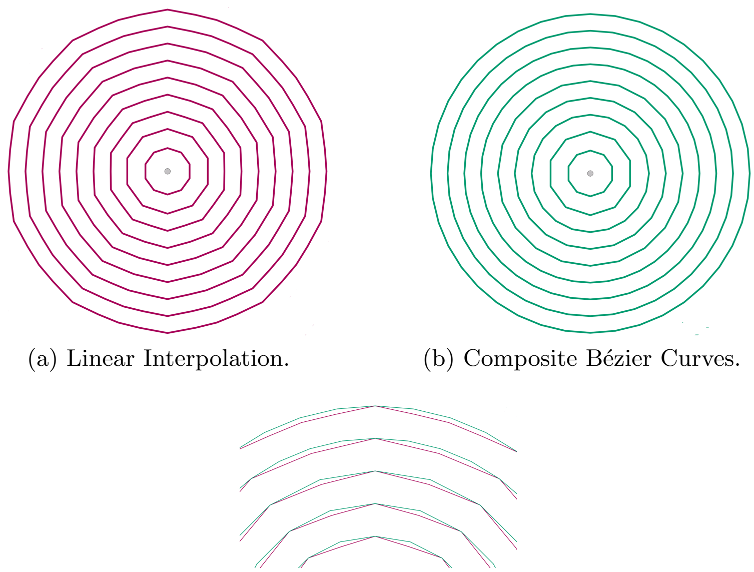

(a) Linear Interpolation. (b) Composite Bézier Curves.

(c) Fire spread comparison.

Fig. 7: Basic ideal case propagation results when linear interpolation (a) and *CBC* interpolation (b) are used in FARSITE. Comparison of the results obtained by both methods (c).

hectares during the first 15 minutes, reaching 100 hectares in less than one hour. The final burned area was 1,697 ha. This final area is shown in Figure 8 with the orange shape. The yellow shape corresponds to one intermediate perimeter, which has been used as the initial perimeter for the experiments reported in this section. This intermediate perimeter corresponds to the area burned by the fire until July 17, 2022 at 16:30. This scenario has been chosen for being characterized by intricate terrain and a complex Wildland-Urban Interface (WUI). This scenario allows to introduce complexities that were absent in the ideal settings, aiming to replicate scenarios frequently encountered in wildfire management, where urban and wildland areas intersect, contributing to heterogeneous landscapes. Figure 9 shows the simulations results in terms of area burned when executing FARSITE including its basic linear interpolation scheme (Figure 9a) and including the *CBC* method (Figure 9b). As it can be observed, the *CBC* method exhibits a better capacity to represent the behavior of wildfires in a more accurate manner, particularly in areas featuring a Wildland-Urban Interface (WUI) and intricate terrain, like the framed area of the fire. FARSITE method tends to underestimate fire spread in those contexts, emphasizing the limitations of conventional linear interpolation in capturing the nuances of complex fire dynamics. The framed area within Figure 9 is characterised for its terrain complexity. The curve perimeter description allows the *CBC* method to recognize accelerated fire propagation, providing a more accurate representation of fire behavior in this area. On the contrary, linear interpolation method fails to capture this nuanced behavior, resulting in a discrepancy in the representation of fire spread.

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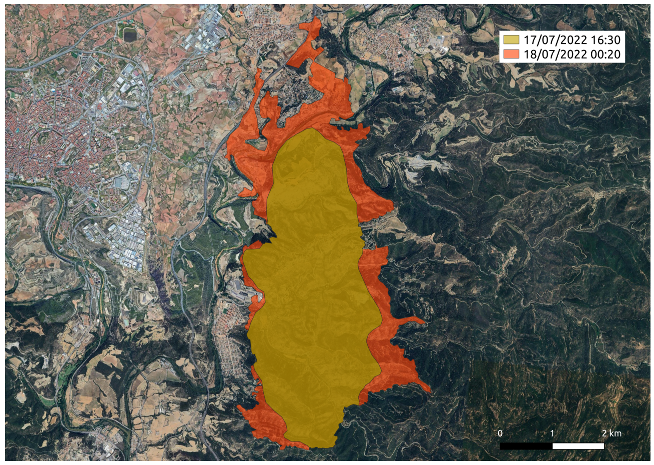
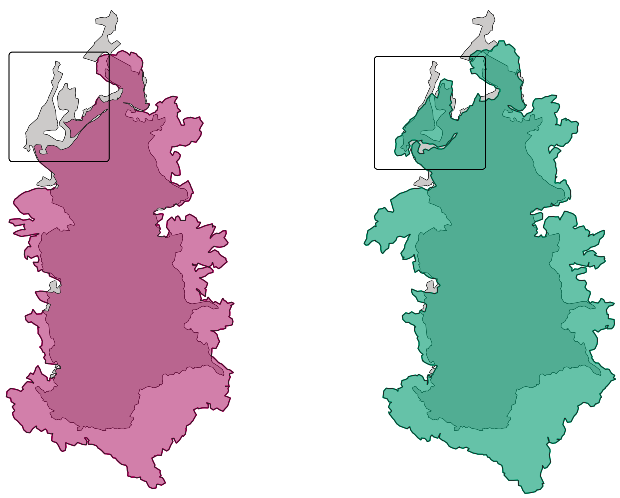


Fig. 8: Initial perimeter (yellow shape) an final burned area (orange shape) of the *Pont de Vilomara* fire.



(a) (b)

Fig. 9: Final simulated burned area using linear interpolation (red shape) (a) and final simulated burned area using the proposed CBS method (green shape) (b). The grey shape corresponds to the final real burned area

The detailed visual analysis of this framed area (Figure 10) pinpoints specific areas where the CBC method excels during the rediscretization stage. The impact of the CBC method on the normal vector that describes the inertial direction of the fire becomes evident. The curve perimeter description afforded by CBC enables the simulator to discern faster fire propagation in strategic areas, a subtlety not effectively captured by the original method. This knowledge, in-

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troduced during rediscretization, propagates between iterations, illustrating the significant influence of the *CBC* method on normal vectors and subsequent fire spread.

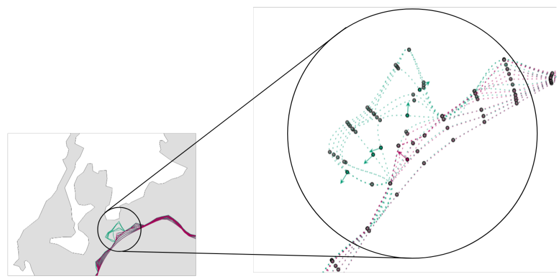

Fig. 10: Detailed aspects of the perimeter evolution when applying linear interpolation (red) and the *CBC* method (green)

5 Conclusions

In this work, an alternative interpolation method for rediscretizing the front of a wildfire in *EWP*-based simulators is proposed. In particular, the proposed approach is based on *Composite Bézier Curves* (*CBC*), which captures more precisely the smooth and circle aspect of the forest fire shape than classical linear interpolation method. The proposed interpolation method has been included in FARSITE, a well-known *EWP*-based simulator, in order to compare its performance compared to the original linear interpolation scheme. The *CBC* method generates fire front that exhibits superior realism compared to the traditional FARSITE interpolation method. This improvement arises for the *CBC* method capacity in capturing intricate fire dynamics, with a particular emphasis on challenging terrains such as Wildland-Urban Interfaces (*WUIs*). In these areas, where urban and wildland environments intersect, accurate simulations are paramount. The adaptability of *CBC* ensures a nuanced depiction of fire behavior, allowing for more effective decision-making in wildfire management and mitigation. The importance of this precision is underscored in *WUIs*, where complex landscapes demand accurate simulations to develop strategies that safeguard both human settlements and natural ecosystems. The successful integration of *CBC* marks a significant step forward in addressing the unique challenges posed by intricate terrains, providing a valuable tool for enhancing the realism and efficacy of wildfire simulations in complex environments.

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