New On-Line Algorithms for Modelling, Identification and Simulation of Dynamic Systems Using Modulating Functions and Non-Asymptotic State Estimators: Case Study for a Chosen Physical Process

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Abstract. The paper presents an advanced application of computation methodology with complicated algorithms and calculation methods dedicated to optimal identification and simulation of dynamic processes. These models may have an unknown structure (the order of a differential equation) and unknown parameters. The presented methodology uses non-standard algorithms for identification of such continuous-time models that can represent linear and non-linear physical processes. Typical approaches, presented in the literature, most often utilize discrete-time models. However, for the case of continuous-time differential equation models, in which both, the parameters and the derivatives of the output variable are unknown, the solution is not easy. In the paper, for the solution of the identification task, the convolution transformation of the differential equation with a special Modulating Function will be used. Also, to be able to properly simulate the behaviour of the process based on the obtained model, the exact state integral observers with minimal norm will be used for the reconstruction of the exact value of the initial conditions (not their estimate). For multidimensional process case, with multiple control signals (many inputs), additional problems arise that make continuous identification and observation of the vector state (and hence simulation) impossible by the use of the standard methods. Application of the above-mentioned methods for solving this problem will be also presented. Both algorithms, for the parameter identification and the state observation, will be implemented on-line in two independent but cooperating windows that will simultaneously move along the time axis. The presented algorithms will be tested using data collected during the heat exchange process in an industrial glass melting installation.

Keywords: Complex algorithms · Multidimensional systems · Process identification · Modulating functions · State observers · Glass forehearth

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1 Introduction

Many industrial processes can be locally approximated by linear models, described by ordinary differential equations. The parameters of these models are most often obtained based on performed identification experiments and because of process non-linearity, can be utilized only in a neighbourhood of specified operating points. However, performing active identification experiments is often impossible due to technological reasons, e.g. in many installations step changes of control signals, during identification experiments, could significantly deteriorate product quality. What is more, complete information about the process is not always available.

Passive identification methodology was widely discussed in the literature, especially for chemical processes, e.g. [1]. On-line implementation of such algorithms can be found in [2]. However, in most cases, discrete models are utilized, e.g. [3]. For the qualitative analysis of processes with distributed parameters, it is common to use the Computational Fluid Dynamics (CFD) approach. It enables precise simulation of glass melting phenomena, however requires a lot of computing power. Hence, in works [4] and [5] procedures of model reduction based on the Proper Orthogonal Decomposition (POD) method are presented. Another, simpler approach, also using Partial Differential Equations (PDE), utilizes the Heat Transfer Equation modelling the glass conditioning process in forehearths, e.g. [6], [7]. In engineering, linear time invariant (LTI) continuous models with lumped parameters are most popular. However, after reduction, in many cases the multidimensionality of such models should be taken into account, i.e. the fact that the process output depends on many inputs (many control signals) - Multi Input-Single Output model (MISO).

In some processes operating points are changed very often, that makes the necessity of changing the process model, which has to be once more re-identified. Hence, establishing of a passive identification algorithm for MISO models, that would be fast and accurate, as well as universal, is an important research goal. This research topic was considered in the previous works of the authors. In the paper [8] the special approach for the identification of MISO models was proposed by the assumption of the existence of several separate, internal, low order SISO sub-models, whose local outputs are unknown (only the main output is measured). The special iteration procedures for identification of each sub-model gave good results, but the algorithm occurred to be a bit complicated and computation time consuming.

In this paper, a new methodology for the identification of MISO models, without local sub-models, is presented. However, the high order of the main model has to be assumed. Identification of the parameters of high-order continuous differential equation is not easy, because only the output $y(t)$ is measured and the values of derivatives $y^{(i)}(t)$ are unknown. To solve this problem, an application of non-standard methods for parameter identification is proposed. A differential equation is transformed by its convolution with modulating functions.

Modulating Function Method (MFM) is the only identification method that leads to the optimal identification of parameters, without introducing any es-

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timation or approximations at any stage of the differential equation transformation and during optimization calculations. Other methods used in the continuous system identification, e.g. by approximation of the input/output functions via orthogonal polynomials [9] introduce immediately methodological errors by definition. Additionally, the exact initial state observation is utilized to enable the model identification and hence the accurate simulation of the process output. The algorithm has been extended with the procedure of properly selecting new operating points, which gave very good modelling results and prediction of the process output values. The presented algorithms was tested using data collected during the glass conditioning process. The process takes place in a long channel, called glass forehearth, which is the final part of the glass melting installation.

The paper is organised as follows. In Sections 2 and 3 theoretical basis of the Modulating Functions Method (MFM) and the Exact Integral State Observers are explained. Developed identification procedure is described in Section 4. Chosen industrial process of glass conditioning is briefly described in Section 5. Obtained simulation results are presented in Section 6. Section 7 draws conclusion.

2 MISO model optimal identification

Linear Tine Invariant (LTI) Multi Input Single Output (MISO) system with K inputs is given as (1):

$$\sum_{i=0}^{n} a_i y^{(i)}(t) = \sum_{k=1}^{K} \sum_{j=0}^{m_k} b_{kj} u_k^{(j)}(t) = \sum_{j=0}^{m_1} b_{1j} u_1^{(j)}(t) + \ldots + \sum_{j=0}^{m_K} b_{Kj} u_K^{(j)}(t). \quad (1)$$

Functions $y^{(i)}$, $u_1^{(j)}, \ldots, u_K^{(j)}$ are the inputs and output derivatives given on the interval $[t_0, T_{ID}]$. There are n output derivatives and m_k derivatives for the k-th input, where $m_k \leq n$, $\forall k = 1, \ldots, K$. Parameters a and b should be identified. The inputs u and the output y can be measured. A deep discussion about continuous-time systems identification can be found in the paper [10] and for MISO systems in [11].

2.1 Modulating Functions Method

Modulating Functions Method (MFM) was developed by M. Shinbrot [12]. Theoretical fundamentals of the method can be found in [13]. It utilizes the rule of integrating by parts. Left and right hand sides of (1) are convoluted with the known modulating function ϕ. This function should meet specified conditions:

- ϕ is supposed to have a compact support of width h (closed and bounded),
- $\phi \in C^{n-1}[0, h]$,
- $\phi^{(i)}(0) = \phi^{(i)}(h) = 0$ for $i = 0, \ldots, n-1$,
- $y * \phi = 0 \Rightarrow y = 0$ on the interval $[t_0 + h, T_{ID}]$.

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In the described method, the Loeb and Cahen functions:

$$\phi(t) = t^N (h-t)^M \tag{2}$$

were used. Utilizing the properties:

$$y_i(t) = a_i \int_{-\infty}^{\infty} y^{(i)}(\tau)\phi(t-\tau)d\tau = a_i \int_0^h y(t-\tau)\phi^{(i)}(\tau)d\tau \stackrel{\text{def}}{=} a_i y_i(t), \tag{3}$$

$$u_{kj}(t) = b_j \int_{-\infty}^{\infty} u_k^{(j)}(\tau)\phi(t-\tau)d\tau = b_j \int_0^h u_k(t-\tau)\phi^{(j)}(\tau)d\tau \stackrel{\text{def}}{=} b_j u_{kj}(t), \tag{4}$$

the set of new known functions $y_i(t), u_j(t)$ in the interval $[t_0+h, T_{ID}]$ is obtained. These functions should be stored in a computer memory. Then, the differential equation (1) can be transformed into the algebraic (5) with the same parameters:

$$\sum_{i=0}^{n} a_i y_i(t) = \sum_{j=0}^{m_1} b_{1j} u_{1j}(t) + \ldots + \sum_{j=0}^{m_K} b_{Kj} u_{Kj}(t) + \epsilon(t). \tag{5}$$

The term ϵ represents a difference between two sides of the equation. It can be treated as an identification performance index (6) in the Equation Error Method (EEM) for identification of the parameters in the equations (1) and (5):

$$\epsilon(t) = \boldsymbol{c}^T(t)\boldsymbol{\theta} = [y_0(t), \ldots, y_n(t), -u_{10}(t), \ldots, -u_{1m_1}(t), \ldots, -u_{K0}(t), \ldots, -u_{Km_K}(t)] \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b_1} \\ \vdots \\ \boldsymbol{b_K} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{a}, \boldsymbol{b_1}, \ldots, \boldsymbol{b_K}$ are the column vectors of suitable dimensions $n+1, m_1+1, \ldots, m_K+1$, $\boldsymbol{\theta} \in R^{n+m_1+\ldots+m_K+K+1}$.

Minimization problem is typically solved using the least squares method. In [14] a different approach was proposed. Minimization problem is stated in the function space $L^2[t_0+h, T_{ID}]$ as:

$$\min_{\boldsymbol{\theta}} J^2 = \min \|\epsilon(t)\|^2_{L^2[t_0+h,T]} = \min \|\boldsymbol{c}(t)^T\boldsymbol{\theta}\|^2_{L^2}. \tag{7}$$

The linear constraint $\boldsymbol{\eta}^T\boldsymbol{\theta} = 1$ is introduced to avoid the trivial solution. The norm in (7) can be written down as an inner product in the space L^2:

$$J^2 = \langle \boldsymbol{c}^T(t)\boldsymbol{\theta}, \boldsymbol{c}^T(t)\boldsymbol{\theta}\rangle_{L^2} = \boldsymbol{\theta}^T\langle \boldsymbol{c}(t), \boldsymbol{c}^T(t)\rangle\boldsymbol{\theta} = \boldsymbol{\theta}^T\boldsymbol{G}\boldsymbol{\theta}. \tag{8}$$

The square real and symmetric Gram matrix $\boldsymbol{G}$ is given as:

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{YY} & \boldsymbol{YU}_1 & \ldots & \boldsymbol{YU}_K \\ \boldsymbol{U}_1\boldsymbol{Y} & \boldsymbol{U}_1\boldsymbol{U}_1 & \ldots & \boldsymbol{U}_1\boldsymbol{U}_K \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{U}_K\boldsymbol{Y} & \boldsymbol{U}_K\boldsymbol{U}_1 & \ldots & \boldsymbol{U}_K\boldsymbol{U}_K \end{bmatrix}, \tag{9}$$

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where:
$\boldsymbol{YY}(i,j) = \langle y_i, y_j \rangle$ and $i = 1 \ldots n$, $j = 1 \ldots n$,
$\boldsymbol{YU_k}(i,j) = -\langle y_i, u_{kj} \rangle$ and $k = 1 \ldots K$, $i = 1 \ldots n$, $j = 1 \ldots m_k$,
$\boldsymbol{U_kY}(i,j) = -\langle u_{ki}, y_j \rangle$ and $k = 1 \ldots K$, $i = 1 \ldots m_k$, $j = 1 \ldots n$,
$\boldsymbol{U}_k\boldsymbol{U}_l(i,j) = \langle u_{ki}, u_{lj} \rangle$ and $k = 1 \ldots K$, $l = 1 \ldots K$, $i = 1 \ldots m_k$, $j = 1 \ldots m_l$.
The matrix G is created by the inner products in L^2 of the $c(t)$ elements, e.g.:

$$\langle y_i, u_j \rangle = \int_{t_0+h}^{T_{ID}} y_i(\tau) u_j(\tau) d\tau. \tag{10}$$

The optimal vector $\boldsymbol{\theta}$, that minimizes the value of J, can be obtained using the Lagrange multiplier technique:

$$\min_{\boldsymbol{\theta}} J^2 = \min_{\boldsymbol{\theta}} L = \min_{\boldsymbol{\theta}} (\boldsymbol{\theta}^T \boldsymbol{G} \boldsymbol{\theta} + \lambda [\boldsymbol{\eta}^T \boldsymbol{\theta} - 1]) \tag{11}$$

as:

$$\boldsymbol{\theta}^0 = \frac{\boldsymbol{G}^{-1}\boldsymbol{\eta}}{\boldsymbol{\eta}^T \boldsymbol{G}^{-1} \boldsymbol{\eta}}. \tag{12}$$

3 Exact integral state observers with minimal norm

In dynamic system theory, for every linear system (1) which describes input-output dependences for a MISO system, one can find corresponding to (1) description of the system in the state space, with the state variable $x(t)$ and the output variable $y(t)$:

$$\begin{aligned} \dot{x}(t) &= \boldsymbol{A}x(t) + \boldsymbol{B}u(t), \qquad x(t_0) = x_0 \\ y(t) &= \boldsymbol{C}x(t), \end{aligned} \tag{13}$$

where:

$$\forall t \geq t_0 : x(t) \in R^n, u(t) \in R^r, y(t) \in R^l,$$

and the corresponding state, control and observation real matrices are: $\boldsymbol{A}_{n \times n}$, $\boldsymbol{B}_{n \times r}$, $\boldsymbol{C}_{l \times n}$ and consist of parameters a_i, b_i from (1) or (5).

In many cases the state vector $x(t)$ is not measured. Only the system output $y(t)$ can be measured. The calculation of $x(t)$ is not easy because the matrix $\boldsymbol{C}$ is not square and hence it is not invertible. The reconstruction of the state vector $x(t)$ for a chosen time t, e.g. $t = t_0$ is well known problem in the control theory. The knowledge of $x(t_0)$ is very important for proper simulation of the real value of the output variable $y(t)$ for the given matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$. To this end, one can use so called state estimators given by differential equations similar to (1), like Kalman Filter or Luenberger observers.

In contrast to differential estimators, the exact state observers have the structure of two integrals and can exactly reconstruct the state of a linear system. The described integral observers guarantee obtaining the real value of the observed state for the observation interval T_{OB}. The theory of optimal observers with minimal norm was described in [15].

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In the paper, two types of observers are utilized. The initial state observer (14) allows obtaining the initial conditions for the identified models when a new operating point is determined. For $t_0 = 0$, the below equation is given:

$$x(0) = \int_0^{T_{OB}} \overline{\boldsymbol{G_1}}(t) y(t) dt + \int_0^{T_{OB}} \overline{\boldsymbol{G_2}}(t) u(t) dt, \tag{14}$$

where:

$$\boldsymbol{M}_0 = \int_0^{T_{OB}} e^{\boldsymbol{A}'\tau} \boldsymbol{C}' \boldsymbol{C} e^{\boldsymbol{A}\tau} d\tau,$$

$$\overline{\boldsymbol{G_1}}(t) = \boldsymbol{M}_0^{-1} e^{\boldsymbol{A}'t} \boldsymbol{C}',$$

$$\overline{\boldsymbol{G_2}}(t) = \boldsymbol{M}_0^{-1} \left[\int_t^{T_{OB}} e^{\boldsymbol{A}'\tau} \boldsymbol{C}' \boldsymbol{C} e^{\boldsymbol{A}\tau} d\tau \right] e^{-\boldsymbol{A}t} \boldsymbol{B}.$$

The final state observer, given as (15), is used for simulation of the model output in the subsequent simulation intervals:

$$x(T_{OB}) = \int_0^{T_{OB}} \boldsymbol{G_1}(t) y(t) dt + \int_0^{T_{OB}} \boldsymbol{G_2}(t) u(t) dt, \tag{15}$$

where:

$$\boldsymbol{G}_1(t) = e^{\boldsymbol{A}T_{OB}} \boldsymbol{M}_0^{-1} e^{\boldsymbol{A}'t} \boldsymbol{C}',$$

$$\boldsymbol{G}_2(t) = e^{\boldsymbol{A}T_{OB}} \boldsymbol{M}_0^{-1} \left[\int_0^{t} e^{\boldsymbol{A}'\tau} \boldsymbol{C}' \boldsymbol{C} e^{\boldsymbol{A}\tau} d\tau \right] e^{-\boldsymbol{A}t} \boldsymbol{B}.$$

It allows to obtain the state value $x(t_j)$ at the end of j-th interval of width T_{OB}. The equation for the system state value at the end of each interval for the moving window version is given as (16):

$$\begin{aligned} x(t_j) = & \int_{t_j - T_{OB}}^{t_j} \boldsymbol{G_1}(T_{OB} - t_j + t) y(t) dt + \\ & + \int_{t_j - T_{OB}}^{t_j} \boldsymbol{G_2}(T_{OB} - t_j + t) u(t) dt, \end{aligned} \tag{16}$$

where the successive time moments are:

$$t_j = p + T - (p \quad \textbf{\textit{modulo}} \quad T) + (j-1) \cdot T, \quad j = 1, 2, 3, \ldots$$

and p is the current operating point.

4 Adaptive identification method

As it was mentioned previously, the described adaptive methodology assumes that the non-linear system can be linearised near a selected operating point. The operating points p are defined for time moments in which input and output

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signals are almost unchanging functions of time, which can be verified by checking if the signal variance is small enough in the defined interval. After determining that the state of the process is steady and the operating point p_1 can be defined, the initial model of the process is identified for the n_{start} intervals assuming zero initial condition.

In case of finding another operating points, the new model is identified and a squared difference between the simulated output of this model and the real system output for the last $n_{reident}$ intervals is calculated and compared with the previous difference, based on the last valid model. If the obtained difference is less than the previous one, the model is updated. If not, then the current version of the model is upheld. The new value of the initial state, needed for the simulation procedure, is obtained with the use of the previously presented initial state observer, whereas the final value of the state in the last interval, essential for the future system output simulation, is determined using the final state observer. It is additionally assumed, that the correlation between the system inputs and output should be greater than the threshold value tr_{corr} for at least one interval among those used in the identification procedure. It prevents from obtaining inaccurate linear models.

The first identification window is significantly longer than the window used for the re-identification procedure. It results from the fact that changes of the input and output signals are rather small in the first intervals and longer signals are needed to obtain a sufficiently accurate model. In the next steps, the identification window is shorter, which allows to obtain the identified model faster, and at the same time causes that the prediction follows the real system output. The developed algorithm is presented in details in the form of Algorithm 1. The marker *empty* in the description means that the first model is still before the identification procedure

It is worth noting that the MFM does not require zero initial condition. This advantage of the method is utilized, when the system signals preceding the new operating point are used in the identification algorithm. The state observers are used only for simulation purposes when the performance index (17), defined as an integral of the squared difference between the real system output and the simulated model output, is calculated:

$$E(t_0, t_{end}) = \int_{t_0}^{t_{end}} (y(t) - y_{sim}(t))^2 dt. \tag{17}$$

The state-space matrices, whose elements were obtained with the MFM, used in the simulation procedures have the form (18):

$$\boldsymbol{A} = \underset{(n\times n)}{\begin{bmatrix} 0 & \dots & 0 & -\frac{a_0}{a_n} \\ 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & 0 & -\frac{a_{n-2}}{a_n} \\ 0 & \dots & 1 & -\frac{a_{n-1}}{a_n} \end{bmatrix}}, \quad \boldsymbol{B} = \underset{(n\times K)}{\begin{bmatrix} \frac{b_{10}}{a_n} & \dots & \frac{b_{K0}}{a_n} \\ \vdots & \vdots & \vdots \\ \frac{b_{1n-1}}{a_n} & \dots & \frac{b_{Kn-1}}{a_n} \end{bmatrix}},$$

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$$\underset{(1\times n)}{\boldsymbol{C}} = \big[\,\big[0\ldots 1\big]\,\big], \quad \underset{(1\times K)}{\boldsymbol{D}} = \big[0\ldots 0\big]. \tag{18}$$

Algorithm 1 Identification and output simulation procedure.

Step 1. Set the current interval counter $j = 1$.

Step 2.

if operating point was found in the last n_{start} intervals **and** *empty*(current model) **then**
Go to **Step 3**.
else if operating point was found in the last $n_{reident}$ intervals **and** not *empty*(current model) **then**
Go to **Step 4**.
else
Go to **Step 5**.
end if

Step 3.

if $j \geq n_{start}$ **and** input-output correlation $\geq tr_{corr}$ **then**
Perform the identification procedure assuming zero initial condition for the n_{start} intervals to obtain the initial model.
end if

Go to **Step 5**.

Step 4.

if $j \geq n_{reident}$ **and** input-output correlation $\geq tr_{corr}$ **then**
Perform the identification procedure for the $n_{reident}$ intervals to obtain the new model.
end if

Calculate the performance index $E_{current}$ for the current model.
Calculate the performance index values for the new models (defined for different parameters) obtained in the last $n_{reident}$ intervals using the initial state observer.
Select the least value for the obtained models $E_{reident}$.

if $E_{reident} < E_{current}$ **then**
Update the current model parameters and save the new operating point.
end if

Go to **Step 5**.

Step 5.

if not *empty*(current model) **then**
Calculate the current state value using the final state observer.
Perform the future output simulation.
end if

Increment the interval counter j.
Go to **Step 2**.

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5 Process description

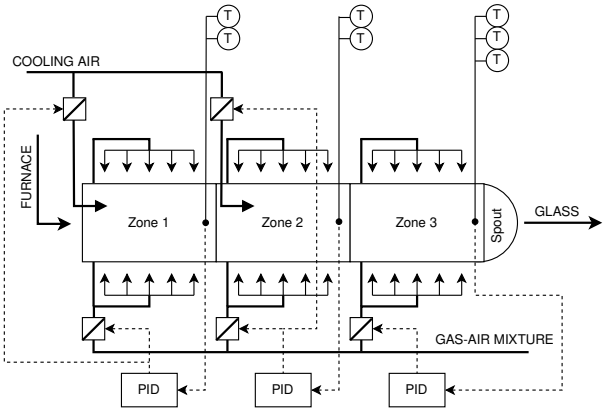

Fig. 1: Forehearth control system

The glass conditioning process involves phased in cooling of a molten glass smelt in order to obtain desired chemical and physical properties. The process is conducted in glass forehearths. Fig. 1 presents an example glass forehearth installation. The forehearth is a long channel divided into several zones. In the first two of them, the glass can be cooled down or heated. The zone controllers adjust signals for cooling dampers and gas burners. In the last zone, there are only gas burners installed. Typically the desired temperature in each zone depends on the type of currently produced container and should be stabilised with an accuracy of at least 1°C. Each forehearth zone is controlled regardless of neighbouring ones.

The on-line identification problem for this installation is associated with many difficulties. The lack of reliable information about the current glass pull rate (total weight of glass containers produced per day) seem to be the most problematic. In most glass factories this parameter is not measured. It can be evaluated only in steady states, when glass gobs fall into a forming machine and parameters of the machine are fixed. Transition between two steady states involves multiple glass pull rate changes, which causes disturbances visible as temperature fluctuations. In the following experiments, the dynamic model of the last zone will be identified. The linear MISO model have two inputs:

- temperature in the previous forehearth zone,
- gas-air mixture pressure,

so the corresponding state space matrix $\boldsymbol{B}$ has two columns. Both component models have the common matrix $\boldsymbol{A}$, which follows directly from the algorithm presented in Subsection 2.1.

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6 Experimental results

Simulation experiments were performed for two sets of data collected from the real glass forehearth installation. The glass pull rate and temperature set point values were noticeably changed for these sets.

The linear constraint vectors used in the MFM procedure have the suitable length depending on the rank of identified systems. The values of the parameters that were used during the experiments are presented in Table 1.

Table 1: Identification procedure coefficients.

Parameter	Description	Value
$\boldsymbol{\eta}$	Linear constraint vector	[1...1]
T	Single interval width	250
n_{start}	Initial identification intervals	8
$n_{reident}$	Intervals for model re-identification	4
$T_{OB_{FIN}}$	Final state observer window width	500
$T_{OB_{INIT}}$	Initial state observer window width	1000
tr_{corr}	Input-output correlation threshold	0.5

The first experiment was performed for the glass pull rate changing from 82 t/24h to 60 t/24h. It caused that the model delay for the first input varied from 260 s to 319 s. In the second case, the glass pull rate was changed in the range 62 t/24h to 55 t/24h and the corresponding delay values fluctuated from 313 s to 337 s. The input signals are presented in Figures 2 and 4 accordingly.

The experimental results are presented in Tables 3 and 5. Figures 3 and 5 show the predicted system output in both cases. Alternating blue and yellow backgrounds denotes the intervals in which subsequent models were applied for the system output prediction. The first green intervals concern the measurements needed for obtaining the initial model, when the system output could not be predicted. The same information are presented in Tables 2 and 4. Parameter *Ident. time* in Tables 2 and 4 concerns the time window used for the identification procedure, while the parameter *Sim. time* refers to the intervals when the identified model was used for the system output prediction. Subsequent operating points are depicted as dotted lines in Figures 3 and 5.

7 Conclusion

In the paper an original application of non-standard optimal method for identification of differential equation parameters was presented. For the purpose of checking the quality of the obtained models, an on-line simulation is performed. In order for the simulation to be correct and to guarantee that the models are accurate at different operating points, a non-standard precise method of identification (observation) of the initial and final states is used. The described

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Table 2: Model properties - 1. experiment

Model nr	Op. point	Ident. time $[s]$	Sim. time $[s]$	MFM parameters		
				N	M	$h[s]$
1	p_1	102-2000	2000-2250	3	4	50
2	p_2	1250-2250	2250-2500	3	4	50
3	p_2	1500-2500	2500-2750	3	4	50
4	p_2	1750-2750	2750-5750	3	4	100
5	p_3	4750-5750	5750-8000	3	4	100

Table 3: Identified model parameters - 1. experiment

Model nr	Parameters						
	a_0	a_1	a_2	b_{10}	b_{11}	b_{20}	b_{21}
1	62.86e−6	10.82e−3	1	141.61e−6	–	247.02e−6	–
2	92.77e−6	11.31e−3	1	119.82e−6	–	153.06e−6	–
3	469.26e−6	45.62e−3	1	694.11e−6	5.73e−3	1.52e−3	309.69e−6
4	321.73e−6	49.84e−3	1	337.46e−6	−3.54e−3	1.59e−3	9.01e−3
5	276.74e−6	44.73e−3	1	203.20e−6	16.20e−3	1.57e−3	11.11e−3

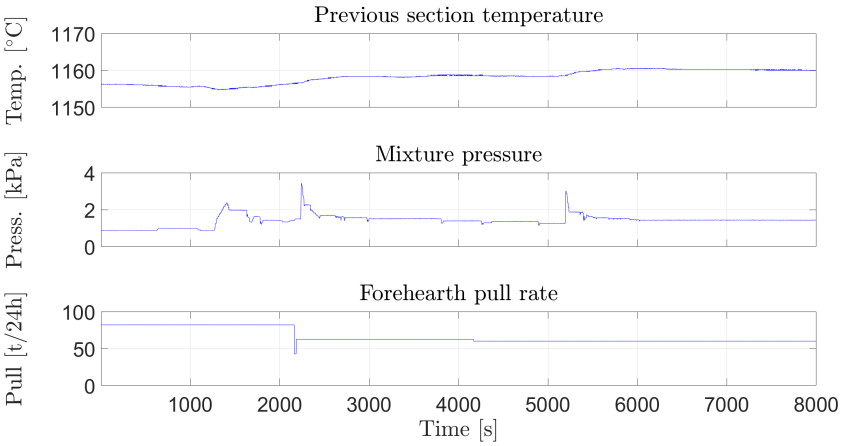

Fig. 2: System inputs and glass pull rate - 1. experiment

Table 4: Model properties - 2. experiment

Model nr	Op. point	Ident. time $[s]$	Sim. time $[s]$	MFM parameters		
				N	M	$h[s]$
1	p_1	102-2000	2000-4500	5	6	100
2	p_2	3500-4500	4500-4750	5	6	100
3	p_2	3750-4750	4750-9500	3	4	50

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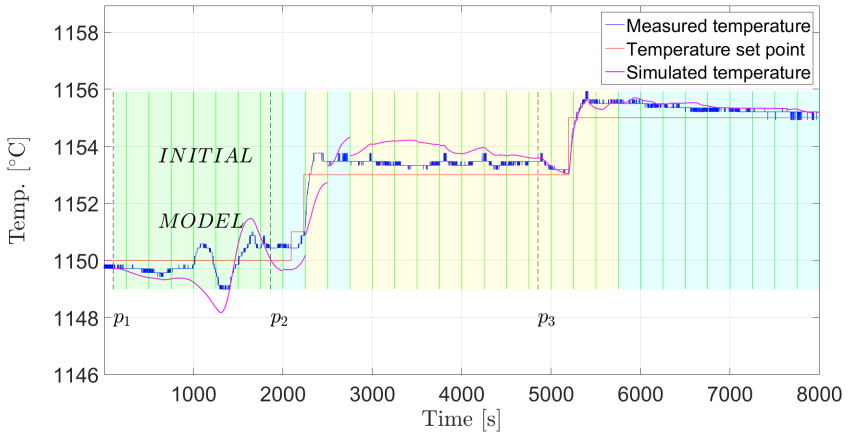

Fig. 3: Simulation results - 1. experiment

Table 5: Identified model parameters - 2. experiment

Model nr	Parameters							
	a_0	a_1	a_2	a_3	b_{10}	b_{11}	b_{20}	b_{21}
1	8.23e−6	2.6e−3	12.47e−3	1	141.61e−6	–	247.02e−6	–
2	6.13e−6	3.67e−3	15.93e−3	1	6.65e−6	−114.32e−6	110.12e−6	848.23e−6
3	168.80e−6	10.68e−3	1	–	65.79e−6	–	466.39e−6	–

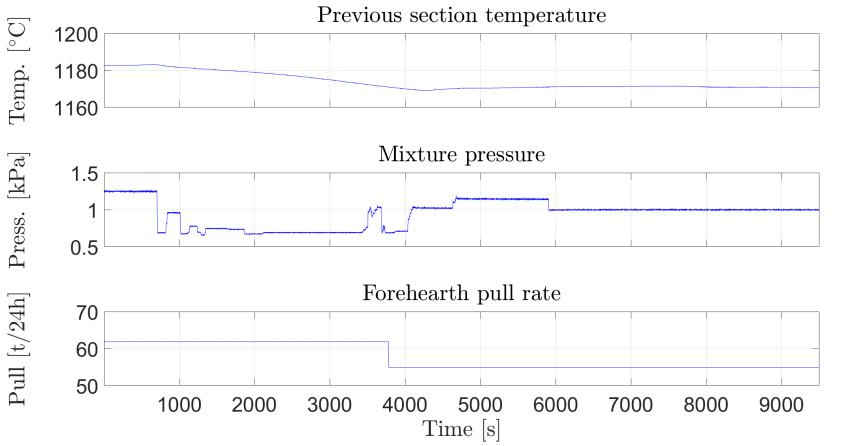

Fig. 4: System inputs and glass pull rate - 2. experiment

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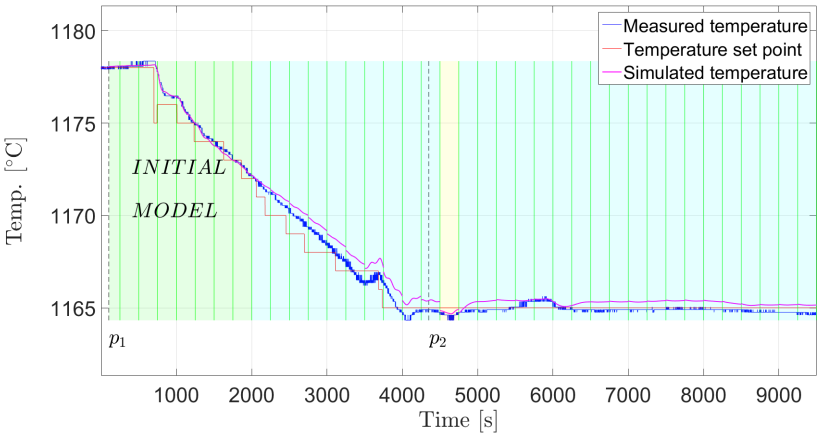

Fig. 5: Simulation results - 2. experiment

parameters identification method using the MFM and the non asymptotic exact state observers allows to model the glass forehearth installation dynamics. Conducted experiments gave very satisfying results. The linear system output was very close to the real system one for both cases. The mean square error between the real system and the simulated output was equal 0.2841 for the first experiment and 0.2028 in the second case. The developed procedure in its current form can be used for wide variety of problems, e.g. PID controller tuning or feed forward control. For the purpose of implementation of the designed algorithms and their testing, an extensive programming environment with many modules was created. The packages were written in the Matlab language, suitable for rapid prototyping, and after automatic translation into C++, it would take tens of thousands of source code lines. It can be used in real computer control systems.

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References

1. Rivas-Perez, R., Sotomayor-Moriano, J., Pérez-Zuñiga, G., Soto-Angles M. E.: Real-Time Implementation of an Expert Model Predictive Controller in a Pilot-Scale Reverse Osmosis Plant for Brackish and Seawater Desalination. Applied Sciences **9**(14), 2932 (2019), doi: 10.3390/app9142932
2. Gough, B.P., Eng, P., Matovich, D.: Predictive-adaptive temperature control of molten glass. In: IEEE Industry Applications Society Dynamic Modeling Control

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To cite this paper please use the final published version:
DOI: 10.1007/978-3-030-77970-2_22

Applications for Industry Workshop, pp. 51–55. Vancouver, BC, Canada, (1997), doi: 10.1109/DMCA.1997.603511
3. Wang, Q., Chalaye, G., Thomas, G., Gilles, G.: Predictive control of a glass process. Control Engineering Practice **5**(2), 167–173 (1997), doi: 10.1016/S0967-0661(97)00223-2
4. Huisman, L., Weiland, S.: Identification and model predictive control of an industrial glass-feeder. IFAC Proceedings Volumes **36**(16), 1645–1649 (2003), doi: 10.1016/S1474-6670(17)34996-0
5. Astrid, P.: Reduction of process simulation models: a proper orthogonal decomposition approach. PhD thesis. Technische Universiteit Eindhoven, 2004, doi: 10.6100/IR581728.
6. Kharitonov, A., Henkel, S., Savodny, O.: Two Degree of Freedom Control for a Glass Feeder. In: Proceedings of the European Control Conference 2007. Kos, Greece, July 2-5, 2007, pp. 4079–4086, doi: 10.23919/ECC.2007.7068450
7. Malchow, F., Sawodny, O.: Model based feedforward control of an industrial glass feeder. Control Engineering Practice **20**(1), 62–68 (2012), doi: 10.1016/J.CONENGPRAC.2011.09.004
8. Byrski, W., Drapała, M., Byrski, J.: An Adaptive Identification Method Based on the Modulating Functions Technique and Exact State Observers for Modeling and Simulation of a Nonlinear Miso Glass Melting Process. International Journal of Applied Mathematics and Computer Science **29**(4), 739–757 (2019), doi: 10.2478/amcs-2019-0055
9. Sinha, N.K., Rao, G.P.: Identification of Continuous-Time Systems, Springer (1991), doi: 10.1007/978-94-011-3558-0
10. Rao, G., Unbehauen, H.: Identification of continous-time systems. IEE Proceedings - Control Theory and Applications **153**(2), 185–220 (2006), doi: 10.1049/ip-cta:20045250
11. Rao, G., Diekmann, K., Unbehauen, H.: Parameter estimation in large scale interconnected systems. IFAC Proceedings Volumes **17**(2), 729–733 (1984), doi: 10.1016/S1474-6670(17)61058-9
12. Shinbrot, M.: On the analysis of linear and nonlinear systems. Transactions of the American Society of of Mechanical Engineers, Journal of Basic engineering **79**, 547–552 (1957)
13. Preisig, H., Rippin, D.: Theory and application of the modulating function method–I. Review and theory of the method and theory of the spline-type modulating functions. Computers & Chemical Engineering **17**(1), 1–16 (1993), doi: 10.1016/0098-1354(93)80001-4
14. Byrski, W., Byrski, J.: The role of parameter constraints in EE and OE methods for optimal identification of continuous LTI models. International Journal of Applied Mathematics and Computer Science **22**(2), 379–388 (2012), doi: 10.2478/v10006-012-0028-3
15. Byrski, J., Byrski, W.: A double window state observer for detection and isolation of abrupt changes in parameters. International Journal of Applied Mathematics and Computer Science **26**(3), 585–602 (2016), doi: 10.1515/amcs-2016-0041

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